Gravitational Absorption According to the Hypotheses of Le Sage and Majorana

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According to kinetic models of gravitation such as Le Sage's and Majorana's, it should be possible to reduce the gravitational attraction between two bodies by the use of material shields. It is usually supposed that Majorana's theory would only predict this effect when the shield is placed between the two bodies, and that Le Sage's theory would predict the existence of this effect in the case of an external shield. This paper presents a quantitative analysis of both theories leading to the conclusion that their predictions are always the same, and that a reduction of gravitational force will always occur whenever straight lines drawn from the test body cut two material bodies.

1. Introduction

Since Newton's time, many authors have proposed mechanical models to explain gravitational forces (Woodward, 1972). Huygens and Leibniz attempted to account for the inverse square law by supposing that "empty" space was full of particles travelling around the gravitating bodies. Newton himself attempted to explain gravitation by several ether models (Aiton, 1969; Hawes, 1968; Rosenfeld, 1969), and at one time he thought that a corpuscular model proposed by Fatio de Duillier (Gagnebin, 1949) would be able to explain all features of these forces. Later he gave up these attempts, and as a result of misinterpretations of his famous "*hypotheses non fingo*," most followers of Newton in the 18th century supposed that one should not attempt to explain gravitational forces. Georges-Louis Le Sage (1784), however, proposed a theory very similar to Fatio's that became famous and gave rise to many other analogous hypotheses in the 19th century. In the early 20th century Hugo von Seeliger (1909), Kurt Bottlinger (1912) and Quirino Majorana (1919, 1920) proposed a new kind of model, assuming that all bodies emit in all directions particles (or waves) of a special type that produce the gravitational forces. These authors emphasised that their theories would imply partial absorption of the gravitational force by matter (Martins, 1999). Theories such as Fatio's or Le Sage's, however, also lead to the same consequence. Both Le Sage's and Majorana's theories belong to the general kind of *kinetic theories of gravitation* (Taylor, 1876). This paper will refer to Le Sage's and Majorana's theories, but the considerations presented here also apply to most similar models.

According to both Le Sage's and Majorana's theories, the gravitational attraction between two bodies is produced by the action of high-speed, invisible

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particles travelling through space. There is no action-at-a-distance in the proper sense, according to these theories: the gravitational force is reduced to local exchange of momentum between the high-speed particles and matter. The interaction of the particles with matter is very weak, however, such that the particles traverse the whole Earth without suffering much absorption.

There are several variant forms of these theories. Some of them replace the particles by waves and introduce different auxiliary hypotheses. For the sake of generality, let us call "gravitational rays" the particles, or waves, or whatever one fancies, that produce the gravitational effects. The distinguishing feature of theories following Le Sage's hypothesis is that material bodies *do not produce* gravitational rays: the whole of space is full of gravitational rays coming from all directions, and material bodies can only change and/or produce absorption of this cosmic background of rays. On the other hand, theories that follow Majorana's hypothesis assume that all material bodies *produce* gravitational rays, besides being able to change and/or produce absorption of the gravitational rays reaching them.

Is it possible to devise experiments that could distinguish between these two kinetic theories of gravitation? Their basic hypotheses are so different that one would expect that they would lead to many conflicting predictions. Majorana himself thought that it was possible to distinguish between the two theories in experiments concerning gravitational absorption; and Radzievskii and Kagalnikova (1960) attempted to prove that Russell's objection against Majorana's theory does not hold when this theory is replaced by a modern version of Le Sage's theory. This paper will show, however, that the forces computed according to both hypotheses are the same, and therefore force measurements (or any other consequence depending only on dynamic effects) cannot be used to choose one of them and to reject the other.

2. Majorana's Analysis and Experiment

In his second series of experiments concerning the absorption of gravitation, Majorana tried to decide whether gravity was due to something emitted from the Earth (his own hypothesis), or something coming to the Earth from space (such as Le Sage's corpuscles). He supposed that in the first case the weight of the test body would be decreased by a screen placed *between* the Earth and the test body, but not if the screen were placed *above* the test body. In the second case, the converse would be true.

> Let us suppose two bodies A and B attracting each other. According to the first model [Majorana's hypothesis], when one puts a third body C between them, the original attractive force would be diminished, because some of the particles travelling between A and B would be absorbed by C. In the case of the second model [Le Sage's hypothesis], the attraction of A towards B is explained as the reciprocal protection or shielding action of these masses against the collisions of the energetic particles that come from distant places of the universe, from all directions. If the third body C were a shield *external* to both masses A and B, it would produce a reduction of the attractive

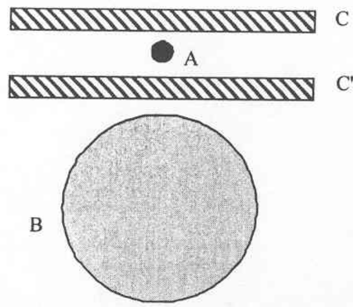

Fig. 1 – When the Earth *B* attracts a test body *A* placed between two thick plates *C* and *C'* the weight of this body should decrease, due to gravitational absorption. Majorana claimed that, according to Le Sage's hypothesis (gravitational rays coming from space) only the plate above the test body should produce gravitational absorption, and that, according to his own hypothesis (gravitational rays emitted by the Earth) only the plate below the test body should produce gravitational absorption.

> force between them, because some of the particles would be captured by C. One may also see that even when the shield is not closed this reduction would occur, although in a lesser measure. Therefore, according to Le Sage's hypothesis, even putting the three bodies in the order A B C, this would engender a diminution of the attractive force between A and B; however, this diminution would only occur, according to the first model, if the three bodies are placed in the order A C B (Majorana, 1921-1922, p. 78).

Majorana attempted to choose between the two hypotheses by comparing the weights of a test body when placed above and below a massive lead shield.

Suppose that *B* is the Earth and *A* is a test body (Fig. 1). According to Le Sage's theory, the gravitational force acting upon *A* is produced by gravitational rays coming from all directions of space. The Earth reduces the flux of upward gravitational rays reaching *A*, and the excess of downward gravitational rays produces the resultant force acting upon *A*—its weight.

According to this hypothesis, we would expect that a thick material plate *C* put above *A*, besides attracting *A*, will also reduce its weight because it will act as a gravitational shield, reducing the flux of gravitational rays coming from space and pushing *A* toward *B*. On the other hand, according to Le Sage's hypothesis, we would expect that a similar plate put in position *C'*, between *A* and the Earth, will attract *A* and increase its weight, but will not decrease the force produced by the Earth, because it will not reduce the flux of gravitational rays coming from space and reaching *A*.

Conversely, according to Majorana's hypothesis, we would expect that when the plate is put between *A* and *B* (position *C'*) its gravitational absorption will decrease the force produced by the Earth upon *A*, but no effect should exist when the plate is in position *C*.

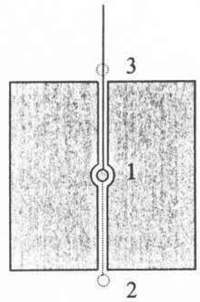

Fig. 2 – In his attempt to choose between Le Sage's and his own hypotheses, Majorana compared the weight of a test body in three positions: at the centre of a lead cube (1), below the cube (2) and above it (3).

To check the hypotheses, Majorana measured the weight of a small test body when it was (1) at the centre of a lead cube; (2) 5 cm below the cube; and (3) 5 cm above the cube (Fig. 2).

The test body was a lead sphere weighing 1.274 kg. The sides of the lead cube, built of lead bricks, measured 95 cm, and its weight was 9,616 kg. In a series of ten measurements, Majorana observed that when the test body was at the centre of the lead cube its weight suffered a reduction amounting to 0.00201 mg, with a standard deviation of 0.00010 mg (Majorana 1921-1922, p. 144). Notice that the standard deviation is about 10^{-10} of the mass of the test body. Majorana was unable to measure the mass of the sphere with this precision. He could only measure very small mass *changes*.

The gravitational attraction of the lead cube, computed according to the Newtonian theory of gravitation, was about 0.217 mg—that is, about 100 times the weight change observed when the test body was at the centre of the cube (Majorana, 1921-1922, p. 222). Therefore, if there were no gravitational absorption, the test body would suffer equal weight changes when it was placed above and below the cube: its weight would *increase* by about 0.2 mg above, and would *decrease* about 0.2 mg below the lead cube.

When Majorana put the test body above the lead cube he observed a weight increase of about 0.2 mg, and when the test body was below the lead cube there was a weight reduction of about 0.2 mg. The two changes were not exactly equal, however. Comparing eight series of measurements, Majorana arrived at the result that when the test body was below the lead cube its weight change was about 0.004 mg larger than when it was above the cube (Majorana, 1921-1922, pp. 223-5, p. 343). That difference was twice the weight reduction of the test body when it was at the centre of the lead cube (0.002 mg).

Majorana's conclusion was that the first hypothesis is the correct one, that is, gravitation is produced by gravitational rays emitted by the attracting bodies, and not by rays coming from space (Majorana 1921/22, p. 79). This experiment is inconclusive, however. Indeed, according to both hypotheses, the change of weight of the body below the cube should be greater than its change of weight above the cube. This can be shown by the following argument.

According to Majorana's own hypothesis (gravitational rays emitted from the Earth), when the test body is above the lead cube (position 3), its weight W would increase by F (the attraction of the cube) and would decrease by f (the absorption of gravitational attraction of the Earth). When the test body is below the lead cube (position 2), its weight W would decrease by F (the attraction of the cube).

According to Le Sage's hypothesis (gravitational rays coming from space), when the test body was above the lead cube, its weight W would increase by F (the attraction of the cube). When the test body was below the lead cube, its weight W would decrease by F (the attraction of the cube) and would decrease by f (the absorption of the gravitational attraction of the Earth).

	Test body above the cube	**Test body below the cube**
Majorana's hypothesis	$W + F - f$	$W - F$
Le Sage's hypothesis	$W + F$	$W - F - f$

Suppose that $F = 200\ \mu$ g and $f = 4\ \mu$ g, as in Majorana's experiment. In this case, the changes of weight would be:

	Test body above the cube	**Test body below the cube**
Majorana's hypothesis	196	–200
Le Sage's hypothesis	200	–204

In both cases, therefore, the change of weight with the test body below the cube should be greater than with the test body above the cube. Majorana's test could not distinguish between the two hypotheses.

3. Comparison between the Two Theories

In the analysis described above, Majorana assumed that a plate between the test body and the Earth would decrease the weight of the body only according to Majorana's own hypothesis, and that a plate above the test body would decrease the weight of the body only according to Le Sage's hypothesis. Majorana's conclusion was shown above to be wrong. Now let us discuss these very assumptions which seem so "natural", but which are, nevertheless, wrong.

Let us consider the following situation (Fig. 3): Two bodies A and B are inside a thick spherical shell S. The resultant force of the shell upon body A is null, according to the Newtonian theory of gravitation. The shell will act, however, as a partial gravitational shield, according to Le Sage's hypothesis, because according to that hypothesis the gravitational force acting upon A is produced by the gravitational rays coming from space, and inside the shell there

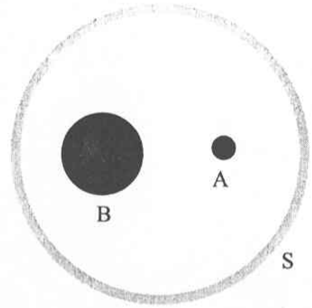

Fig. 3 – According to Le Sage's hypothesis, when two bodies *A* and *B* are inside a thick spherical shell *S*, the force produced by the shell upon them will be null. However, the gravitational force between *A* and *B* will decrease, because the shell will reduce the flux of gravitational rays coming from the outer space, which produce the force between the two bodies.

will be a smaller density of gravitational rays than outside it. Consequently, *B* will produce a smaller force upon *A*.

According to Majorana's hypothesis, on the other hand, it seems that the force produced by *B* upon *A* cannot be influenced by the spherical shell *S*, because the force acting on *A* is produced by gravitational rays emitted by *B* and the shell does not have any influence on that emission. A more careful analysis of the situation, however, shows that according to Majorana's hypothesis the force acting upon *A* should be smaller when the shield *S* is introduced.

Indeed, when *A* alone is inside the spherical shell and body *B* does not exist, the resultant force acting upon it is null. However, when *B* is introduced inside the shell, it will produce a twofold effect (Fig. 4). First, its gravitational rays will produce a force upon *A*. Second, *B* will act as a partial gravitational shield as regards *S*, because some of the gravitational rays emitted by *S* will pass through *B* before reaching *A*. Therefore, the force produced by the shell upon *A* in the direction of *B* will be smaller than the force it produces upon *A* in the opposite direction. Adding this effect to the attraction produced by *B*, we see that the resultant force acting upon *A* is smaller than the force produced by *B* alone. The shell *S* is not acting as a screen, but nevertheless it does reduce the force between *A* and *B*.

So, both according to Le Sage's hypothesis and according to Majorana's hypothesis, the external shield will reduce the force between *A* and *B*.

The above analysis is sufficient to show that a comparison between the two hypotheses is not as straightforward as it might seem at a first sight. Of course, this qualitative analysis cannot establish whether the effect of the spherical shell has the same value according to both hypotheses. It is necessary to *compute* the forces to compare them.

4. Le Sage's Theory in One Dimension

Let us compute the effects of gravitational absorption in the cases of Majorana's hypothesis (that is, gravitational rays emitted by material particles) and Le Sage's hypothesis (that is, gravitational rays coming from space). First

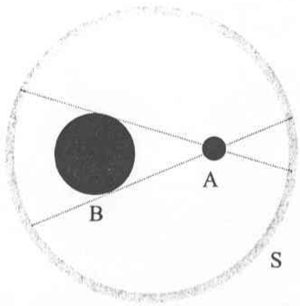

Fig. 4 – According to Majorana's hypothesis, when two bodies *A* and *B* are inside a thick spherical shell *S*, the gravitational force between them should be the same, because the shell cannot affect the emission of gravitational rays by the two bodies. However, *B* will act as a partial shield of the rays coming from the shell towards *A*, and therefore there will be a non-null resultant force produced by the shell upon *A*.

let us consider this issue in the one-dimensional case, and then in the three-dimensional situation.

As we are interested here only in the computation of *forces* between bodies, let us adopt a simple model where gravitational rays are not reflected: they can only traverse matter or undergo absorption. Let as also assume that there is a single kind of gravitational ray, carrying a momentum p. More complicated models, with a spectrum of rays and considering reflection, diffusion and transformation of gravitational rays would follow similar lines.

First, according to Le Sage's hypothesis, space is full of gravitational rays travelling in all directions. Let us call Φ_0 the momentum flux $p(dN/Sdt)$ of these rays in empty space. Consider a single slab of matter with surface S, thickness L and density ρ (Fig. 5).

When the rays that are travelling from the left to the right pass through the slab of matter they suffer partial absorption, and the flux changes from Φ_0 to $\Phi_1 = \Phi_0\, e^{-h\rho L}$. Of course, the rays travelling in the opposite direction suffer an equal change.

The absorption of gravitational rays produces a force equal to $p\, dN'/dt$, where p is the momentum of each ray and dN'/dt is the rate of absorption of rays. If there were only rays travelling from the left to the right, they would produce a force F on the matter slab equal to

$$F = p\frac{dN'}{dt} = S\left(\Phi_0 - \Phi_1\right) = S\Phi_0\left(1 - e^{-h\rho L}\right) \tag{4.1}$$

Let us introduce in (1) the absorption factor $\mu = 1 - e^{-h\rho L}$ (approximately equal to $h\rho L$) and the equation becomes:

$$F = S\Phi_0\left(1 - e^{-h\rho L}\right) = S\Phi_0\mu \tag{4.2}$$

Of course, there is an opposite force produced by the absorption of rays travelling in the opposite direction, and the net force upon the matter slab is null.

Let us now consider two matter slabs A and B (Fig. 6).

Suppose the bodies have different densities and thickness. Each one will therefore have a different absorption factor $\mu = 1 - e^{-h\rho L}$. Let μ_A be the absorption factor of body A, and μ_B the absorption factor of body B.

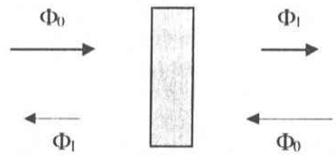

Fig. 5 – According to Le Sage's hypothesis, each material body is traversed by gravitational rays coming from all directions. The flux of gravitational rays must decrease in traversing the material body, because of gravitational absorption.

The following relations will hold:

$$\Phi_1 = \Phi_0 (1 - \mu_A) \quad (4.3)$$

$$\Phi_2 = \Phi_1 (1 - \mu_B) = \Phi_0 (1 - \mu_A)(1 - \mu_B) \quad (4.4)$$

$$\Phi_3 = \Phi_0 (1 - \mu_B) \quad (4.5)$$

$$\Phi_4 = \Phi_1 (1 - \mu_A) = \Phi_0 (1 - \mu_A)(1 - \mu_B) \quad (4.6)$$

The force produced upon A by the gravitational rays that are travelling from the left to the right is:

$$F_A^+ = p\frac{dN'}{dt} = S(\Phi_0 - \Phi_1) = S\Phi_0\mu_A \quad (4.7)$$

The force produced upon A by the gravitational rays that are travelling from the right to the left is:

$$F_A^- = p\frac{dN''}{dt} = S(\Phi_3 - \Phi_4) = S\Phi_0 (1 - \mu_B)\mu_A \quad (4.8)$$

Therefore the net force acting upon A will be:

$$F = F_A^+ - F_A^- = S\Phi_0\mu_A - S\Phi_0 (1 - \mu_B)\mu_A = S\Phi_0\mu_A\mu_B \quad (4.9)$$

The net force F_B acting upon B has an equal value and opposite direction, as may be easily seen—therefore, the law of action and reaction holds in this case. Notice that μ_A and μ_B have roles similar to the masses of the attracting bodies in Newton's gravitational law. As $\mu = 1 - e^{-h\rho L} \cong h\rho L$, and since the mass M of each plate is $M = \rho LS$, we have $\mu \cong hM/S$.

According to this model, the gravitational force between two bodies is due to two circumstances: first, to the existence of a cosmic background of gravitational rays; second, to the partial absorption of gravitational rays by matter. Each body attracts the other one because it acts as a partial screen for the cosmic background of gravitational rays.

Let us now consider the case of three matter slabs A, B and C (Fig. 7). In this case, the force produced upon A by the gravitational rays that are travelling from the left to the right is the same as in the former case:

$$F_A^+ = p\frac{dN'}{dt} = S(\Phi_0 - \Phi_1) = S\Phi_0\mu_A \quad (4.10)$$

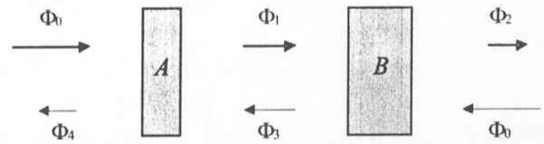

Fig. 6 – A simple one-dimensional model helps to understand the gravitational interaction between two bodies *A* and *B*, according to Le Sage's hypothesis. The gravitational flux Φ_0 coming from the outer space will undergo successive reductions as it traverses the two bodies. There will be a resultant force acting upon the body if it absorbs a non-null momentum from the gravitational rays.

The force produced upon A by the gravitational rays that are travelling from the right to the left is:

$$F_A^- = p\frac{dN''}{dt} = S(\Phi_6 - \Phi_5) = S\Phi_0(1-\mu_C)(1-\mu_B)\mu_A \tag{4.11}$$

Therefore the net force acting upon A will be:

$$\begin{gathered} F_A = F_A^+ - F_A^- = S\Phi_0\mu_A - S\Phi_0(1-\mu_C)(1-\mu_B)\mu_A \therefore \\ \therefore F_A = S\Phi_0\mu_A(\mu_B + \mu_C - \mu_B\mu_C) \end{gathered} \tag{4.12}$$

If only A and B existed, the net force would be:

$$F_{AB} = S\Phi_0\mu_A\mu_B \tag{4.13}$$

On the other hand, if only A and C existed, the net force would be:

$$F_{AC} = S\Phi_0\mu_A\mu_C \tag{4.14}$$

Therefore,

$$F_A \neq F_{AB} + F_{AC} \tag{4.15}$$

Notice that the net force may be represented in two ways:

$$F_A = S\Phi_0\mu_A(\mu_B + \mu_C - \mu_B\mu_C) = F_{AB} + F_{BC} - \mu_B F_{AC} \tag{4.16}$$

$$F_A = S\Phi_0\mu_A(\mu_B + \mu_C - \mu_B\mu_C) = F_{AB} + F_{BC} - \mu_C F_{AB} \tag{4.17}$$

We might say that when B is introduced between A and C it produces an attraction upon A, and at the same time decreases the attraction between A and C (that is, B acts as a partial gravitational shield because it is between A and C). That is the interpretation of (4.16).

However, as the equation of the net force acting upon A is completely symmetrical as regards B and C, it might also be interpreted the other way around: when C is introduced close to the interacting bodies A and B, it produces an attraction upon A, and at the same time decreases the attraction between A and B, because it acts as a partial screen relative to the cosmic background of gravitational rays. That is the interpretation of (4.17).

The net force upon B can be easily computed in a similar way:

$$F_B^+ = S(\Phi_1 - \Phi_2) = S\Phi_0(1-\mu_A)\mu_B \tag{4.18}$$

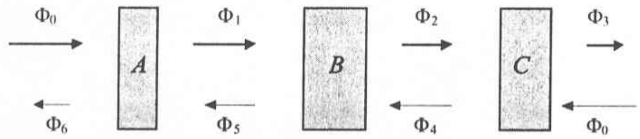

Fig. 7 – According to the simple one-dimensional model of Le Sage's hypothesis it is possible to calculate the resultant force acting upon *A* when two nearby bodies *B* and *C* act as partial shields of the flux of gravitational rays. The computation shows that the effect is not additive, that is, the force acting upon *A* when both *B* and *C* are present is smaller than the sum of the forces produced separately by *B* and *C*.

$$F_B^- = S(\Phi_4 - \Phi_5) = S\Phi_0 (1-\mu_C)\mu_B \tag{4.19}$$

$$F_B = F_B^+ - F_B^- = S\Phi_0\mu_B\left[(1-\mu_A)-(1-\mu_C)\right] = S\Phi_0(\mu_C - \mu_A)\mu_B \tag{4.20}$$

The net force acting upon C will be:

$$F_C^+ = S(\Phi_2 - \Phi_3) = S\Phi_0(1-\mu_A)(1-\mu_B)\mu_C \tag{4.21}$$

$$F_C^- = S(\Phi_0 - \Phi_4) = S\Phi_0\mu_C \tag{4.22}$$

$$\begin{gathered} F_C = F_C^+ - F_C^- = S\Phi_0\mu_C\left[(1-\mu_A)(1-\mu_B)-1\right]\therefore \\ \therefore F_C = S\Phi_0\mu_C(-\mu_A - \mu_B + \mu_A\mu_B) \end{gathered} \tag{4.23}$$

The sum of the three forces $F_A + F_B + F_C$ is equal to zero. It is easy to see that $\Phi_3 = \Phi_6$, that is, the net decrease of the flux of gravitational rays is the same in both directions.

What exactly is the force between A and B in this case? If one assumes Le Sage's theory, there is no definite answer to such a question. As a matter of fact, A and B are not acting upon one another: they are acting on and being acted upon by the gravitational rays.

However, if one prefers to describe the interaction as occurring between the material bodies, one might say that there is a force between A and B and that it is not changed by the presence of C:

$$F_{AB} = S\Phi_0\mu_A\mu_B = -F_{BA} \tag{4.24}$$

In that case, it would be necessary to interpret the remaining part of the force acting upon A as due to C:

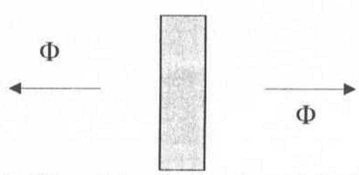

Fig. 8 – According to Majorana's hypothesis each material body is incessantly emitting gravitational rays in all directions.

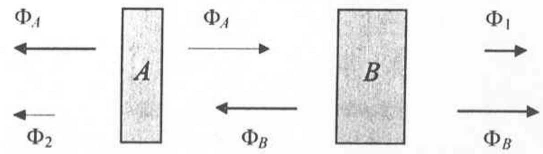

Fig. 9 – A simple one-dimensional model helps to understand the gravitational interaction between two bodies *A* and *B*, according to Majorana's hypothesis. The gravitational fluxes Φ_A and Φ_B emitted by these bodies will undergo a reduction as they traverse the other body. The absorbed momentum will produce a resultant force upon each body. It is necessary to suppose that the momentum carried by each gravitational ray is opposite to its velocity.

$$F_{AC} = S\Phi_0\mu_A\mu_C(1-\mu_B) = -F_{CA} \tag{4.25}$$

This division of the total force acting upon *A* corresponds to the interpretation of *B* as a partial shield of the force between *A* and *C*.

According to the alternative interpretation, one might say that the force between *A* and *B* is changed by the presence of *C*:

$$F_{AB} = S\Phi_0\mu_A\mu_B(1-\mu_C) = -F_{BA} \tag{4.26}$$

In that case, the force between *A* and *C* would be:

$$F_{AC} = S\Phi_0\mu_A\mu_C = -F_{CA} \tag{4.27}$$

According to this interpretation, the force between *B* and *C* would be partially screened by the presence of *A*, too:

$$F_{BC} = S\Phi_0\mu_B\mu_C(1-\mu_A) = -F_{CB} \tag{4.28}$$

Remember, however, that (4.16) and (4.17) are completely equivalent equations and that, from the mathematical point of view, both interpretations lead to the same result.

5. Majorana's Theory in One Dimension

Let us now develop a similar analysis following Majorana's hypothesis. According to that hypothesis, each body is continually emitting gravitational rays in all directions. Let us disregard the cosmic background of gravitational rays that would be produced by that emission.

Consider a single slab of matter with surface *S*, thickness *L* and density ρ (Fig. 8). As a first step let us consider the one-dimensional case, and let us suppose that this body emits gravitational rays with a momentum flux $\Phi = p(dN/Sdt)$ in each direction. This flux will depend on the properties of the body, and it will be approximately proportional to its thickness and its density, when self-absorption is small. Let us suppose that the emitted flux is proportional to a magnitude *M* that we shall call the "active gravitational mass" of the body: $\Phi = kM$.

There is no net force acting upon the slab, because the rate of emission of gravitational rays in both directions is the same.

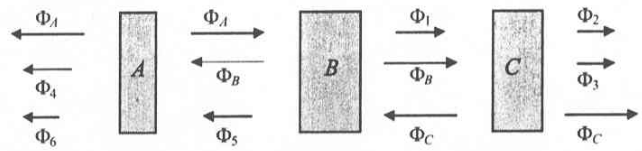

Fig. 10 – According to the simple one-dimensional model of Majorana's hypothesis it is possible to calculate the resultant force acting upon *A* when there are two nearby bodies *B* and *C*. The computation shows that the effect is not additive, that is, the force acting upon *A* when both *B* and *C* are present is smaller than the sum of the forces produced separately by *B* and *C*, because the gravitational rays coming from C to A will be partially absorbed by *B*.

Let us now consider two matter slabs, *A* and *B* (Fig. 9). The gravitational flux emitted by each body is proportional to its active gravitational mass: $\Phi_A = kM_A$ and $\Phi_B = kM_B$.

Suppose again that the bodies have different densities and thickness, and thus that each will have a different absorption factor $\mu = 1 - e^{-h\rho L}$. Let μ_A be the absorption factor of body *A*, and μ_B the absorption factor of body *B*.

When the rays emitted by *A* to the right pass through *B* they suffer partial absorption, and the flux changes from Φ_A to $\Phi_1 = \Phi_A(1 - \mu_B)$. Of course, the rays emitted by *B* that pass through *A* suffer a similar change: $\Phi_2 = \Phi_B(1 - \mu_A)$.

The absorption of gravitational rays produces a force equal to pdN'/dt, where p is the momentum of each ray and dN'/dt is the rate of absorption of rays. According to Majorana's hypothesis, the momentum imparted by the gravitational rays is in a direction opposite to their velocities. Therefore, rays travelling to the right produce a force to the left, and *vice versa*. In what follows, only the absolute value of these forces will be computed.

The force produced upon *A* by the partial absorption of the gravitational rays emitted by *B* is:

$$F_A = p\frac{dN'}{dt} = S(\Phi_B - \Phi_2) = S\Phi_B\mu_A \tag{5.1}$$

The force produced upon *B* by the partial absorption of gravitational rays emitted by *A* is equal to:

$$F_B = p\frac{dN''}{dt} = S(\Phi_A - \Phi_1) = S\Phi_A\mu_B \tag{5.2}$$

But $\Phi_A = kM_A$ and $\Phi_B = kM_B$, therefore:

$$F_A = SkM_B\mu_A \tag{5.3}$$

$$F_B = SkM_A\mu_B \tag{5.4}$$

If these forces obey the law of action and reaction, we must have $F_A = F_B$, and therefore $M_B\mu_A = M_A\mu_B$. Hence $M_B/\mu_B = M_A/\mu_A$, that is, the active gravitational mass M of each body must be proportional to its absorption factor μ. Let us assume that the law of action and reaction is valid, and that $M = k'\mu$. Hence,

$$F_A = F_B = Skk'\mu_B\mu_A \tag{5.5}$$

Now let us consider the case of three bodies A, B and C (Fig. 10). The gravitational flux Φ_A emitted by body A becomes $\Phi_1 = \Phi_A(1 - \mu_B)$ after traversing the body B, and $\Phi_2 = \Phi_A(1 - \mu_B)(1 - \mu_C)$ after passing through body C. The gravitational flux Φ_B emitted by body B becomes $\Phi_3 = \Phi_B(1 - \mu_C)$ after traversing the body C, and $\Phi_4 = \Phi_B(1 - \mu_A)$ after passing through body A. The gravitational flux Φ_C emitted by body C becomes $\Phi_5 = \Phi_C(1 - \mu_B)$ after traversing the body B, and $\Phi_6 = \Phi_C(1 - \mu_B)(1 - \mu_A)$ after passing through body A.

The total force produced upon A will be due to its partial absorption of the gravitational rays emitted by both B and C:

$$F_A = S(\Phi_B - \Phi_4) + (\Phi_5 - \Phi_6) = S\left[\Phi_B + \Phi_C(1-\mu_B)\right]\mu_A \tag{5.6}$$

Replacing Φ_B by $kk'\mu_B$ and Φ_C by $kk'\mu_C$ we obtain:

$$F_A = Skk'(\mu_B + \mu_C - \mu_B\mu_C)\mu_A \tag{5.7}$$

If only A and B existed, the net force would be:

$$F_{AB} = Skk'\mu_A\mu_B \tag{5.8}$$

On the other hand, if only A and C existed, the net force would be:

$$F_{AC} = Skk'\mu_A\mu_C \tag{5.9}$$

Therefore,

$$F_A \neq F_{AB} + F_{AC} \tag{5.10}$$

Notice that the net force acting upon A may be represented in two ways:

$$F_A = Skk'(\mu_B + \mu_C - \mu_B\mu_C)\mu_A = F_{AB} + F_{AC} - \mu_B F_{AC} \tag{5.11}$$

$$F_A = Skk'(\mu_B + \mu_C - \mu_B\mu_C)\mu_A = F_{AB} + F_{AC} - \mu_C F_{AB} \tag{5.12}$$

This result is mathematically equivalent to that obtained under Le Sage's hypothesis, equations (4.16) and (4.17). The interpretation, however, is slightly different. In the case of Majorana's hypothesis, it is more natural to regard B as reducing the force between A and C, because it produces a partial absorption of the gravitational rays emitted by A and by C towards each other. It would be odd to say that C reduces the force between A and B. However, this is just a matter of interpretation. The equation of the net force acting upon A is completely symmetrical as regards B and C, exactly as in the case of Le Sage's model.

6. Le Sage's Theory in Three Dimensions

So, the predictions of the two models are the same, in the one-dimensional case. Does this result hold in real, three-dimensional situations?

Let us suppose that A is a very small test body. According to Le Sage's hypothesis, the gravitational force acting upon this body is the result of differences between the fluxes of gravitational rays coming from different directions (Fig. 11). Consider a cone with its vertex at A, comprising a very small solid

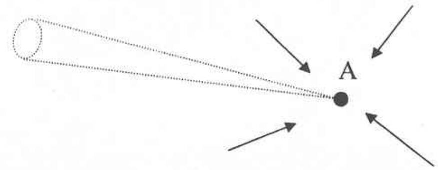

Fig. 11 – In the three-dimensional case, according to Le Sage's hypothesis, each body is acted upon gravitational rays coming from all directions and the resultant force is derived by computing the gravitational flux reaching *A* from an elementary cone, and integrating over all directions.

angle $d\Omega$. The axis of the cone has the direction $\hat{r}$. Suppose that the flow $d\phi$ of gravitational rays reaching the body *A* from direction $\hat{r}$, coming through the cone comprising the solid angle $d\Omega$ is

$$d\phi = f(\hat{r})d\Omega \tag{6.1}$$

The resultant gravitational force acting upon *A* will be proportional to the resultant flow of gravitational rays reaching *A*.

$$\vec{F} = k\iint f(\hat{r})\hat{r}d\Omega \tag{6.2}$$

Let us suppose that *B* is a large body close to *A* (Fig. 12). Let us assume that *B* has a homogeneous composition, that is, a constant density. The form of *B* is arbitrary. The dimensions of *A* are negligible when compared to its distance to *B* and to the dimensions of *B*. Let us compute the force produced by *B* upon *A*, according to Le Sage's hypothesis.

Consider a cone with its vertex at *A*, comprising a very small solid angle $d\Omega$. The axis of the cone has the direction $\hat{r}$. The axis of the cone intersects *B* between the distances r_1 and r_2. These distances are a function of the direction of $\hat{r}$.

Suppose that $d\phi_0 = f_0\, d\Omega$ is the isotropic flow corresponding to the cosmic background of gravitational rays. This is the flow reaching *A* from every direction $\hat{r}$ except those directions that intercept the body *B*. The flow $d\phi$ reaching *A* from directions $\hat{r}$ that intercept the body *B* will be:

$$d\phi = f_0 e^{-h\rho L}d\Omega\,, \tag{6.3}$$

where *L* is the thickness of body *B* traversed by the gravitational rays before they reach body *A*. This thickness is a function of the direction:

$$L = r_1 - r_2 = L(\hat{r}) \tag{6.4}$$

The resultant gravitational force acting upon *A* will be proportional to the resultant flow of gravitational rays reaching *A* from all directions

$$\vec{F} = -k\iint \hat{r}d\phi = -\iint f_0 e^{-h\rho L}\hat{r}d\Omega \tag{6.5}$$

Replacing $e^{-h\rho L}$ by $1 - \lambda(\hat{r})$ and taking into account that $\iint f_0 \hat{r}\; d\Omega = 0$ we obtain:

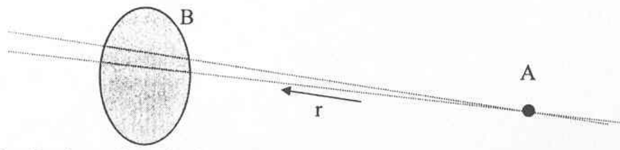

Fig. 12 – According to Le Sage's hypothesis, a test body *A* is drawn towards *B* because the gravitational absorption reduces the flux of gravitational rays coming from *B*. To find the force acting upon *A* it is necessary to compute the reduction of the gravitational flux reaching *A* from each elementary cone passing through *B*.

$$\vec{F} = -\iint f_0\left[1-\lambda(\hat{r})\right]\hat{r}d\Omega = kf_0 \iint \lambda(\hat{r})\hat{r}d\Omega \qquad (6.6)$$

This is a general result that is valid both when *A* is inside *B*, and when it is outside *B*.

Let us now analyse the case of two large bodies *B* and *C* acting upon *A* (Fig. 13). Consider again a cone with its vertex at *A*, comprising a very small solid angle $d\Omega$. The axis of the cone has the direction $\hat{r}$. Depending on the direction, the cone will intersect both *B* and *C*, or only *B*, or only *C*, or none of them. Let $L_B(\hat{r})$ be the length inside body *B* traversed by the axis of the cone, and let $L_C(\hat{r})$ be the length inside body *C* traversed by the axis of the cone. Both quantities depend on the direction $\hat{r}$, and one of them or both may be null in some directions.

Since $d\phi_0 = f_0\, d\Omega$ is the flow corresponding to the cosmic background of gravitational rays, the flow $d\phi$ reaching *A* from directions $\hat{r}$ that intercepts the bodies *B* and *C* will be:

$$d\phi = f_0 e^{-h\rho_B L_B} e^{-h\rho_C L_C} d\Omega \qquad (6.7)$$

Replacing $\exp(-h\rho L)$ by $1 - \lambda(\hat{r})$ we obtain:

$$d\phi = f_0\left[1-\lambda_B(\hat{r})\right]\left[1-\lambda_C(\hat{r})\right]d\Omega \qquad (6.8)$$

The resultant gravitational force acting upon *A* will be proportional to the resultant flow of gravitational rays reaching *A* from all directions

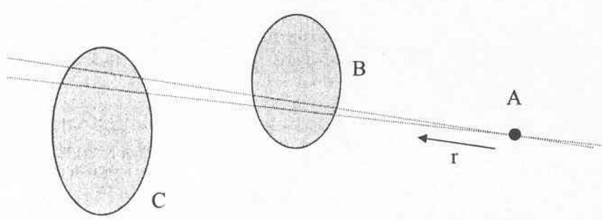

Fig. 13 – According to Le Sage's hypothesis, when there are two bodies *B* and *C* in the same direction, close to *A*, they will both absorb the gravitational rays reaching *A* from that direction. To find the force acting upon *A* it is necessary to compute the reduction of the gravitational flux reaching *A* from each elementary cone. In the case of the rays passing through both *B* and *C* the effect is not additive, and hence the resultant force acting upon A is smaller than the sum of the forces produced by *B* and *C* separately.

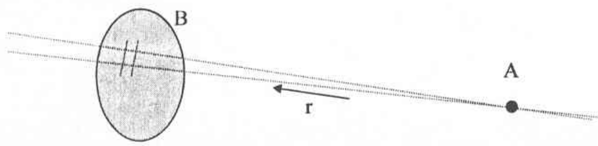

Fig. 14 – In the three-dimensional case, according to Majorana's hypothesis, a body *B* acts upon another body *A* by emission of gravitational rays. However, it is also necessary to take into account that a fraction of these rays are absorbed within the emitting body itself. To find the force acting upon *A* it is necessary to compute the attraction produced by each mass element of *B*, taking into account the reduction of this attraction due to the absorption of gravitational rays inside *B*.

$$\vec{F} = -k \iint \hat{r} d\phi = -k \iint f_0 \left[1 - \lambda_B(\hat{r})\right]\left[1 - \lambda_C(\hat{r})\right] \hat{r}\, d\Omega \tag{6.9}$$

Taking into account that $\iint f_0 \hat{r}\, d\Omega = 0$ we obtain:

$$\vec{F} = kf_0 \left[\iint \lambda_B(\hat{r}) \hat{r} d\Omega + \iint \lambda_C(\hat{r}) \hat{r}\, d\Omega - \iint \lambda_B(\hat{r}) \lambda_C(\hat{r}) \hat{r}\, d\Omega \right] \tag{6.10}$$

The first term is the force that acts upon *A* when only *B* exists. The second integral is the force upon *A* when only *C* exists. The third integral is the effect associated to the shielding of the gravitational rays. The integrand is different from zero only in the directions that intersect both *B* and *C*. As $\lambda_B(\hat{r})$ and $\lambda_C(\hat{r})$ play symmetrical roles in the equation, it is possible to interpret this term as a shielding effect produced by *B* (which is between *C* and *A*) reducing the force between *C* and *A*, or as an "external" shielding effect produced by *C*, reducing the force between *B* and *A*.

If no radius vector drawn from the test body *A* crosses both bodies, the third integral will be null, and the force acting upon *A* will be just the sum of the forces produced by *B* and *C*.

7. Majorana's Theory in Three Dimensions

Let us now consider Majorana's hypothesis. We assume that there is no background flux of gravitational rays. Suppose that the small test body *A* is close to a large body *B*, as in the former hypothesis. Now, each part of body *B* should be regarded as an active source of gravitational rays that are emitted in all directions. It is also necessary to take into account the self-absorption of the gravitational rays inside *B* (Fig. 14).

Let us suppose that the body *B* is homogeneous, with a constant density ρ_B. However, taking into account the whole space around *A*, we may regard the density ρ at any point around *A* to be a function of its radius vector $\vec{r} = r\hat{r}$.

Consider again the cone with its vertex at *A*, comprising a very small solid angle $d\Omega$. The axis of the cone has the direction $\hat{r}$. The mass *dm* encompassed within this cone between the distances *r* and *r* + *dr* is:

$$dm = \rho(\vec{r})\, r^2 d\Omega\, dr \tag{7.1}$$

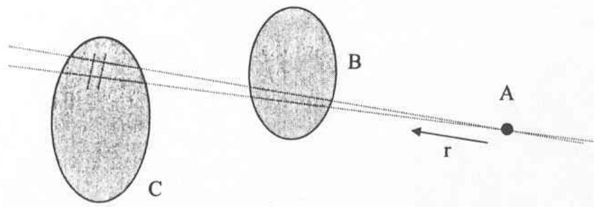

Fig. 15 – According to Majorana's theory, when there are two bodies *B* and *C* in the same direction, close to *A*, they will both emit and absorb gravitational rays towards *A*. To find the force acting upon *A* it is necessary to compute the attraction produced by each mass element of each body upon A, taking into account the absorption of gravitation in both *B* and *C*. In the case of the rays passing through both *B* and *C* the effect is not additive, and hence the resultant force acting upon A is smaller than the sum of the forces produced by *B* and *C* separately.

The axis of the cone intersects B between the distances r_1 and r_2. These distances are a function of the direction of $\hat{r}$. The density is null, for each direction, when $r > r_1$ or $r < r_2$.

If the mass of the body A is M, the gravitational attraction between A and the mass dm encompassed within this cone between the distances r and $r + dr$ is:

$$d\vec{F} = -GMe^{-h\rho(r-r_2)}r^{-2}\hat{r}dm \therefore$$
$$\therefore d\vec{F} = -GMe^{-h\rho(r-r_2)}\rho(\vec{r})\,\hat{r}\,d\Omega\,dr \tag{7.2}$$

This expression is valid whatever the value of r, because when $r > r_1$ or $r < r_2$ the density ρ is null, and therefore the force is also null. The total force acting upon A because of the presence of B is the integral of (7.2) over all space:

$$\vec{F}_B = -GM\iiint e^{-h\rho(r-r_2)}\rho(\vec{r})\,\hat{r}\,d\Omega\,dr \tag{7.3}$$

Keeping the direction $\hat{r}$ constant and varying r, the density is null outside B and it is equal to ρ_B between r_1 and r_2. Therefore, integrating (7.3) over r, we obtain:

$$\vec{F}_B = -(GM/h)\iint\left[e^{-h\rho_B(r_1-r_2)} - 1\right]\hat{r}\,d\Omega \tag{7.4}$$

Replacing $\exp[-h\rho_B(r_1 - r_2)]$ by $1 - \lambda_B(\hat{r})$ we obtain:

$$\vec{F}_B = -(GM/h)\iint\lambda_B(\hat{r})\,\hat{r}\,d\Omega \tag{7.5}$$

Notice that the result has the same form as equation (6.6) obtained according to Le Sage's hypothesis.

Let us now consider the case of two large bodies B and C acting upon A (Fig. 15). According to Majorana's hypothesis, the force produced by B upon A will not be influenced by the body C which is placed outside the region encompassing A and B. The force produced by C upon A, however, is influenced by B, because this body is between them, and there will be a partial absorption of the gravitational rays emitted by C towards A.

Consider once more a cone with its vertex at A, comprising a very small solid angle $d\Omega$. The axis of the cone has the direction $\hat{r}$. Depending on the direction, the cone will intersect both C and B. Let us suppose that the axis of the cone enters body C at a distance r_3 and leaves the body at a distance r_4, with $r_4 > r_3$.

Let us suppose that the body C is homogeneous, with a constant density ρ_C. The gravitational force upon A produced by each small element of C comprised inside the cone and between the distances r and $r + dr$ is:

$$d\vec{F}_C = -GMe^{-h\rho_C(r-r_3)}\left[1-\lambda_B(\hat{r})\right]r^{-2}\,\hat{r}\,dm\;\therefore$$
$$\therefore\; d\vec{F}_C = -GMe^{-h\rho_C(r-r_3)}\left[1-\lambda_B(\hat{r})\right]\rho_C\,\hat{r}\,d\Omega\,dr \qquad (7.6)$$

In some directions the cone does not intercept the body B, and in these cases $\lambda_B(\hat{r}) = 0$.

The total force produced by C upon A is the integral of (7.6) over the volume of C:

$$\vec{F}_C = -GM\iiint e^{-h\rho_C(r-r_3)}\left[1-\lambda_B(\hat{r})\right]\rho_C(\vec{r})\,\hat{r}\,d\Omega\,dr \qquad (7.7)$$

Keeping the direction $\hat{r}$ constant and varying r between r_3 and r_4 we obtain, by integration over r:

$$\vec{F}_C = -(GM/h)\iint\left[e^{-h\rho_C(r_4-r_3)}-1\right]\left[1-\lambda_B(\hat{r})\right]\hat{r}\,d\Omega \qquad (7.8)$$

Replacing $\exp[-h\rho_C(r_4 - r_3)]$ by $1 - \lambda_C(\hat{r})$ we obtain:

$$\vec{F}_C = (GM/h)\iint\lambda_C(\hat{r})\left[1-\lambda_B(\hat{r})\right]\hat{r}\,d\Omega\;\therefore$$
$$\therefore\;\vec{F}_C = (GM/h)\iint\lambda_C(\hat{r})\,\hat{r}\,d\Omega-(GM/h)\iint\lambda_B(\hat{r})\,\lambda_C(\hat{r})\,\hat{r}\,d\Omega \qquad (7.9)$$

The first integral corresponds to the force that would be produced by C upon A if B did not exist. The second integral corresponds to the reduction of the force produced by C upon A because of the partial absorption by B of the gravitational rays coming from C.

Therefore, the total force acting upon A is:

$$\vec{F}_A = \vec{F}_B + \vec{F}_C =$$
$$=(GM/h)\left[\iint\lambda_B(\hat{r})\,\hat{r}\,d\Omega+\iint\lambda_C(\hat{r})\,\hat{r}\,d\Omega-\iint\lambda_B(\hat{r})\,\lambda_C(\hat{r})\,\hat{r}\,d\Omega\right] \qquad (7.10)$$

Notice that the final result is completely symmetrical as regards B and C. If we compare this result with that obtained according to Le Sage's hypothesis in equation (6.10),

$$\vec{F} = kf_0\left[\iint\lambda_B(\hat{r})\hat{r}d\Omega+\iint\lambda_C(\hat{r})\hat{r}\,d\Omega-\iint\lambda_B(\hat{r})\lambda_C(\hat{r})\hat{r}\,d\Omega\right]$$

we see that they are completely equivalent, since they contain exactly the same integrals. Therefore, if the constants in both equations are adjusted so that the forces produced by each body (B and C) upon A are the same in both models, the absorption force will also be equal according to both theories.

8. Final Comments

Let us now return to the situation described at the beginning of this paper. Majorana assumed that his own theory and Le Sage's theory would lead to different absorption effects and that experiments would be able to distinguish between them. That guess was grounded upon loose qualitative analysis, but it went unchallenged up to the present.

Now, according to the quantitative analysis developed above, it becomes clear that when a plate *C* is put above a test body *A*, it will produce no gravitational absorption effect, because no radius vector drawn from *A* will pass through both *B* and *C*. This result is valid according to both Majorana's theory and Le Sage's. In this situation the force acting upon *A* is simply the vector sum of the forces produced separately by *B* and *C*. On the other hand, when the plate is put between the test body and the Earth (*C'*), it will produce a gravitational absorption effect, according to both theories, and the value of this effect is exactly the same, independently of the chosen theory. As both theories lead to the same force effects, no experimental measurement of forces will be able to provide a criterion for choosing between them.

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