WAVE MECHANICS, FROM LOUIS DE BROGLIE TO SCHRÖDINGER: A COMPARISON

Roberto de Andrade Martins

Abstract: Erwin Schrödinger's work on wave mechanics started in late 1925, stimulated by his study of Louis de Broglie's thesis. It is well known that in his initial attempts to formulate a quantum theory of the atom Schrödinger tried to develop a relativistic theory, following de Broglie's ideas, and only afterwards he looked for a non-relativistic wave equation. It is straightforward to derive the wave equation corresponding to de Broglie's phase waves. both in the relativistic and non-relativistic realms. In the case of his relativistic attempt, Schrödinger did indeed follow a simple approach, using de Broglie's theory. In the non-relativistic approach, he attempted to produce an independent derivation of the wave equation, following several different lines, instead of using de Broglie's results in the classical limit. This paper analyses Schrödinger's derivations of the wave equation, showing the differences and similarities between his theory and de Broglie's. It will be shown that, although it is formally possible to derive the wave equation from de Broglie's theory, there is an incompatibility between the two theories: it would be impossible to make any sense of de Broglie's ideas in the case of the rigid rotator, for instance. Schrödinger's approach was, in this sense, independent and incompatible with de Broglie's theory, and it could be easily applied to many different physical situations. This heuristic value of Schrödinger's wave equation is another very important distinction between the two theories, since de

MARTINS, Roberto de Andrade. *Studies in History and Philosophy of Science II*. Extrema: Quamcumque Editum, 2021.

Broglie's theory only led to a single new prediction: the wave behaviour of electrons in diffraction experiments.
Keywords: Schrödinger, Erwin; de Broglie, Louis; wave mechanics; wave equation; quantum mechanics; history of physics

1. INTRODUCTION

The researches of Erwin Rudolf Josef Alexander Schrödinger (1887-1961) on wave mechanics started in late 1925 as a development of his study of the 1924 thesis of Louis-Victor-Pierre-Raymond de Broglie (1892-1987). It is well known that Schrödinger's wave equation can be derived from De Broglie's results, in the classical limit. From this point of view, one might think that Schrödinger's theory seems a mere development of De Broglie's theory. However, can we really accept that conclusion?

This paper will compare some features of De Broglie's and Schrödinger's theories. It is well known that in his initial attempts to formulate a quantum theory of the atom Schrödinger tried to develop a relativistic theory, following de Broglie's ideas, and only afterwards he looked for a non-relativistic wave equation. It is straightforward to derive a wave equation for de Broglie's phase waves both in the relativistic and non-relativistic realms. In the case of his relativistic attempt, Schrödinger did indeed follow a simple approach, using de Broglie's theory. In developing the non-relativistic approach, however, Schrödinger attempted to produce an independent derivation of the wave equation, following several different lines, instead of using de Broglie's results in the classical limit.

I will first discuss the historical influence of De Broglie's work on Schrödinger; then, the early derivations of the wave equation, stressing the differences and similarities between the two theories. It will be shown that, although it is formally possible to derive a wave equation from de Broglie's theory, there is an incompatibility between the two theories: it would be impossible to make any sense of de Broglie's ideas in the case of the rigid rotator, for instance. Schrödinger's approach was, in this sense, independent and incompatible with de Broglie's theory, and it could be easily applied to many different physical situations.[1]

2. DE BROGLIE'S THEORY

Louis de Broglie's theory was first presented in a series of papers published in 1923-1924 (Broglie, 1923a, 1923b, 1923c, 1923d; 1924a, 1924b, 1924c) and in his PhD thesis (Broglie, 1924d; 1925). He took as a starting point the idea that all particles (electrons, light quanta, etc.) underwent some periodical process obeying both the relativistic and quantum energy equations $E=h\nu$ and $E=mc^2$, and used special relativity as the main theoretical tool of his work (Broglie, 1923a).[2]

In the rest frame of the particle, one should have $E_0=m_0c^2=h\nu_0$ and relative to other reference systems, the correct equation should be:

$$E=mc^2=h\nu \tag{1}$$

However, mass increases with speed, and frequency decreases with speed. Therefore, it seemed that the use of $E=mc^2=h\nu$ would lead to a contradiction (Broglie, 1923a, pp. 507-508). After dealing with this difficulty for a while, De Broglie recognised that his theory could only obey the special theory of relativity if he conceived all quanta as extended systems, instead of point particles.

[1] This work was written for presentation at the workshop "Quantum theory: historical studies and cultural implications", held at the Universidade Federal da Paraíba, Campina Grande, Brazil, 15-17 December 2008. A shorter Portuguese version has already been published (Martins, 2010), but this English version is now published for the first time.

[2] A detailed discussion of De Broglie's work can be found in Martins & Rosa, 2014 (in Portuguese).

In his thesis he presented this fundamental idea in a very clear way. According to Maxwell's electromagnetic theory, the energy of any charge (including an electron) is spread in the space around it, although it has a strong energy concentration around a centre. Following this idea, De Broglie regarded the electron as an infinite system (Broglie, 1925, pp. 33-34).

In the rest frame of an electron, its whole (infinite) structure was supposed to be pulsating in synchrony, with a frequency given by $h\nu_0=m_0c^2$. This periodic phenomenon, independent of space, could be described by an equation such as:

$$\Psi_0 = A \operatorname{sen} 2\pi(\nu_0 t_0) \tag{2}$$

Relative to other reference systems, the synchrony of this periodic phenomenon would be lost, of course, due to relativistic effects. The Lorentz transformation of time is:

$$t_0 = \frac{1}{\sqrt{1-\beta^2}}\left(t - \frac{\beta x}{c}\right) \tag{3}$$

where $\beta=v/c$ is the speed of the particle divided by the speed of light.

Applying the Lorentz transformation of time to this pulsation, de Broglie easily showed that the oscillation would transform to a wave, relative to other reference frames, and obtained the speed, frequency and other properties of the wave (Broglie, 1925, pp. 35-36). Replacing t_0 in (2) by (3), we get:

$$\Psi_0 = A \operatorname{sen} 2\pi\left[\nu_0 \frac{1}{\sqrt{1-\beta^2}}\left(t - \frac{\beta x}{c}\right)\right] \tag{4}$$

The general formula for a monochromatic wave travelling in the x direction is:

$$\Psi_0 = A \operatorname{sen} 2\pi\left[\nu\left(t - \frac{x}{V}\right)\right] \tag{5}$$

Comparison of equations (4) and (5) shows that the uniform pulsation of the electron (in its proper reference frame) becomes a monochromatic wave (the “phase wave”) relative to other reference systems; and by identifying the corresponding quantities in both equations, one obtains the frequency ν and speed V of the wave associated to the electron:

$$\nu = \frac{\nu_0}{\sqrt{1-\beta^2}} \qquad (6)$$

$$V = \frac{c}{\beta} = \frac{c^2}{\mathrm{v}} \qquad (7)$$

The electron should have some definite position, of course. Therefore, the uniform infinite wave cannot describe it completely. The moving free electron would be equivalent to an extended system with strong energy concentration around a centre, travelling at a speed v, and at the same time traversed by a monochromatic wave of speed $V=c^2/\mathrm{v}$ and frequency $\nu=mc^2/h$.

A modulated monochromatic wave is mathematically equivalent to a wave group, but conceptually it is quite different, because in the rest frame it does have a single, well-defined frequency, and it shouldn’t spread as it moves.

3. MECHANICS AND OPTICS

De Broglie presented his theory in several different ways. In some of his publications he emphasised the similarities between mechanics and optics (Broglie, 1924a; 1925, pp. 46-53). In the special theory of relativity, the Maupertuis’ principle of least action can be written as:

$$\delta\int_P^Q J_i dx^i = 0 \qquad (8)$$

where J_i are the components of the energy-momentum four-vector.

On the other hand, the relativistic version of Fermat's principle can be written as:

$$\delta\int_P^Q O_i dx^i = 0 \qquad (9)$$

where O_i are the components of the four-vector "universe wave", with components corresponding to the wave number projections and the frequency of the wave.

The analogy between the two principles (Fermat and Maupertuis) and the relation $E=h\nu$ then allowed De Broglie to establish a general relation in four dimensions:

$$O_i = \frac{1}{h} J_i \qquad (10)$$

which contain both $E=h\nu$ and $p=h/\lambda$ as special cases of the relativistic equation (Brown & Martins, 1984).

4. ELECTROMAGNETIC FIELDS

If the electron is moving in an electromagnetic field, its total energy (including the potential energy $e\varphi$) remains constant (Broglie, 1925, p. 60). De Broglie supposed that the frequency of the electron wave should be proportional to the total energy W, and therefore it would also be constant.

$$h\nu = W = \frac{m_0 c^2}{\sqrt{1-\beta^2}} + e\varphi \qquad (11)$$

However, the speed V of the waves and its wavelength would change from place to place, according to a very complex equation (Broglie, 1925, p. 60):

$$V = \frac{W}{p} = \frac{\frac{m_0 c^2}{\sqrt{1-\beta^2}} + e\varphi}{\frac{m_0 \beta c}{\sqrt{1-\beta^2}} + eA_l} = \frac{c}{\beta} \frac{W}{W - e\varphi} \cdot \frac{1}{1 + eA_l/G} \qquad (12)$$

In this equation, the momentum of the electron contains components proportional to the potential vector $\boldsymbol{A}$, following Maxwell's theory (Bork, 1967).

De Broglie did not attempt to apply (11) and (12) to any specific situation. The only case of a bound particle he was able to deal with was the hydrogen atom (Broglie, 1923a, pp. 509-510). He supposed that the centre of the electron obeyed classical mechanics and followed a Kepler path. He assumed that the wave would follow the same classical trajectory. Assuming that the wave should always be in phase with the electron oscillations (and not assuming that the wave should be stationary, as presented in textbooks), he proved that Bohr's quantum rule for the angular momentum $L=pr=nh/2\pi$ was a consequence of his own theory (Broglie, 1925, pp. 62-65).

The only new prediction of De Broglie's theory was electron diffraction (Broglie, 1923b, p. 549; 1925, p. 104), and this was soon confirmed. Experiments with high-energy electrons proved, in a few years, that the wavelength of the electron wave obeyed a relativistic equation, as predicted by De Broglie.

5. THE EINSTEIN CONNECTION

It is usually said that De Broglie's thesis was only accepted due to Albert Einstein's influence upon Paul Langevin (Mehra & Rechenberg, 1982, vol. 1.2, p. 604). This version, grounded upon De Broglie's testimony, is not correct.

Paul Langevin told Einstein about De Broglie's work in July 1924, and on July 27 he asked De Broglie to send a copy of his thesis (before it was approved) to Einstein (Wheaton, 1983, p. 297; Darrigol, 1993, p. 355). However, there was no immediate reaction from Einstein. De Broglie's thesis was presented and approved on November 25. Only on December 16 Einstein wrote letters to Langevin and to Lorentz praising De Broglie's work – "He has lifted a corner of the big veil" (Darrigol, 1993, p. 355; Mehra & Rechenberg, 1982, vol. 1.2, p. 604). On January 13 Langevin wrote a letter to De Broglie, telling him about Einstein's favourable opinion (Wheaton, 1983, p. 297).

At this time Einstein was working on the quantum theory of gases (now called "Bose-Einstein statistics"). In a paper published in February 1925 he remarked that De Broglie's work might help to elucidate the meaning of the new theory (Jammer, 1966, p. 249).

Erwin Schrödinger read Einstein's papers, and they exchanged letters about the quantum theory of gases (Hanle, 1977; 1979). Stimulated by Einstein's reference to De Broglie's work, Schrödinger obtained a copy of the thesis and read it in October 1925[3]. In November 1925 Schrödinger wrote letters to Einstein and to Landé showing that he was very excited with De Broglie's ideas (Moore, 1989, p. 192). He applied De Broglie's theory to gases in a paper he finished in December 1925 (Hegt, 1997, p. 474). However, he found some features of De Broglie's theory difficult to understand or to accept – especially the theory of the hydrogen atom.

On November 23, 1925, Schrödinger presented a seminar on De Broglie's ideas (Moore, 1989, p. 192). At that occasion, Peter Debye remarked that De Broglie's approach was childish and that it was necessary to use a wave equation to describe the wave in three dimensions (Kragh, 1982, p. 157). Schrödinger agreed that the waves should be dealt with in another way, in the case of the hydrogen atom. He also noticed that the waves

[3] According to Heitler (1961, p. 222) many other physicists studied De Broglie for the same reason, but nobody – except Schrödinger – took the idea of waves associated to electrons seriously. See Raman & Forman's analysis of Schrödinger's peculiar attitude towards De Broglie's work (Raman & Forman, 1969).

in nearby Keplerian orbits would produce a distorted wave front, therefore De Broglie's approach was too simplistic.

In December 1925 Schrödinger began his attempts to produce a wave equation from De Broglie's theory and to apply it to the hydrogen atom. Instead of waves following Keplerian orbits he began to think about standing waves in three dimensions, analogous to sound waves in cavities. Quantization should arise as a consequence of the discrete spectrum of standing waves in the atom.

6. SCHRÖDINGER'S RELATIVISTIC WAVE EQUATION

Some decisive steps were made around Christmas, during Schrödinger's stay in Villa Herwig, in the Alps, where he spent two weeks with a mysterious lover (Moore, 1989, pp. 194-195). Schrödinger first tried to produce a relativistic wave equation, following De Broglie's approach (Kragh, 1982, pp. 175-178). This derivation was not published but it can be found in a manuscript, probably written in late December 1925 (Mehra & Rechenberg, 2001, vol. 5.1, pp. 423-430). Let us present a reconstruction of Schrödinger's first derivation of the wave equation (Kragh, 1982, p. 180; Kragh, 1984).

The general wave equation, valid both in classical and relativistic physics, is:

$$\Delta\psi + \left(\frac{2\pi}{\lambda}\right)^2 \psi = 0 \qquad (13)$$

For any monochromatic wave $\lambda = V/\nu$, therefore the general wave equation can also be written as a function of the speed of the wave and its frequency:

$$V^2 \Delta\psi + 4\pi^2 \nu^2 \psi = 0 \qquad (14)$$

In De Broglie's theory, $V = E/p$ where E is the total energy of the electron:

$$E = h\nu = mc^2 - e\phi = (m_0c^2)/(1-\beta^2)^{1/2} - e\phi \qquad (15)$$

If magnetic fields are disregarded, the momentum p is:

$$p=mv=(m_0\beta c)/(1-\beta^2)^{1/2} \qquad (16)$$

Since the total energy of the electron is equal to hν (eq. 15,) it is possible to obtain v=βc as a function of ν. Substituting this result in the equations for E and p one can compute the wave velocity V = E/p as a function of the frequency ν. After some manipulation, the general wave equation (13) becomes:

$$\frac{h^2c^2}{4\pi^2}\Delta\psi + \left[(E - e\varphi)^2 - m_0^2c^4\right]\psi = 0 \qquad (17)$$

This is the so-called "Klein-Gordon equation".

Notice that Schrödinger's derivation of the relativistic wave equation depends only on results that had already been obtained by De Broglie. Indeed, De Broglie himself did also arrive to the same result, independently (Kragh, 1984, p. 1025).

Schrödinger applied this wave equation to the hydrogen atom and obtained *wrong* results for the energy levels (Jammer, 1966, pp. 257-258; Mehra & Rechenberg, 2001, vol. 5.1, pp. 367-368). After struggling for a short time with the relativistic theory he turned to a non-relativistic approach.

7. THE CLASSICAL WAVE EQUATION

Obtaining a wave equation in the classical approximation is much easier than in the relativistic case, and many textbooks present such a derivation. If one accepts the relation $\lambda=h/p$ between momentum and wavelength and applies classical dynamics, one obtains:

$$\lambda = \frac{h}{p} = \frac{h}{m\text{v}} \qquad (18)$$

In classical mechanics, the kinetic energy K is:

$$K = \frac{m\mathrm{v}^2}{2} = E - U \qquad (19)$$

Therefore, the square of wavelength, in the classical limit, is:

$$\lambda^2 = \frac{h^2}{p^2} = \frac{h^2}{m^2\mathrm{v}^2} = \frac{h^2}{2m(E-U)} \qquad (20)$$

The general (classical and relativistic) wave equation is:

$$\Delta\psi + \left(\frac{2\pi}{\lambda}\right)^2 \psi = 0 \qquad (21)$$

Replacing λ^2 in (21) by the expression in (20), one obtains:

$$\Delta\Psi + \frac{8\pi^2 m}{h^2}(E-U)\Psi = 0 \qquad (22)$$

This is the so-called Schrödinger equation independent of time. Therefore, using only classical mechanics and De Broglie's relation $\lambda = h/p$ it is possible to derive the Schrödinger wave equation. Notice that this derivation is very similar to Schrödinger's relativistic derivation shown above, but much simpler. However, Schrödinger did not use this very simple derivation. How did Schrödinger present the wave equation in his early papers?[4]

8. THE WAVE EQUATION IN SCHRÖDINGER'S FIRST PAPER

In his first 1926 paper Schrödinger presented the wave equation as a consequence of the Hamilton-Jacobi approach, in a very abstract way (Schrödinger, 1926a, pp. 361-362; 1929, pp.

[4] I will not discuss here how Schrödinger arrived to his equation. The published papers do not present his ideas as they were really developed (Kragh, 1982, p. 158). What interests me is how Schrödinger chose to publish his results and what could be his motivation for presenting them exactly in this way.

1-2). He introduced an unknown function Ψ and stated that the action *S* could be written as

$$S = K \log \Psi \qquad (23)$$

The Hamiltonian function *H* could therefore be written as:

$$H\left(q, \frac{\partial \mathrm{S}}{\partial \mathrm{q}}\right) = H\left(q, \frac{K}{\Psi}\frac{\partial \Psi}{\partial \mathrm{q}}\right) = E \qquad (24)$$

Schrödinger then stated that in the non-relativistic case, this equation "can always be transformed so as to become a quadratic form (of Ψ and its derivatives) equated to zero" (Schrödinger, 1926a, pp. 361-362; 1929, p. 1), and in the case of the hydrogen atom (a Coulombian field) this would become:

$$\left(\frac{\partial \Psi}{\partial x}\right)^2 + \left(\frac{\partial \Psi}{\partial y}\right)^2 + \left(\frac{\partial \Psi}{\partial z}\right)^2 - \frac{2m}{K^2}\left(E + \frac{e^2}{r}\right)\Psi^2 = 0 \qquad (25)$$

This is just the classical equation of energy conservation $p^2=2m(E-V)$, since the three first terms correspond the squared momentum divided by (*K*/Ψ), according to (24).

Schrödinger than stated (Schrödinger, 1926a, p. 362; 1929, p. 2): "We now seek a function Ψ such that for any arbitrary variation of it the integral of the said quadratic form, taken over the whole co-ordinate space, is stationary [...]". The corresponding equation is:

$$\delta J = \delta \left[\left(\frac{\partial \Psi}{\partial x}\right)^2 + \left(\frac{\partial \Psi}{\partial y}\right)^2 + \left(\frac{\partial \Psi}{\partial z}\right)^2 - \frac{2m}{K^2}\left(E + \frac{e^2}{r}\right)\Psi^2\right] dx.dy.dz = 0 \qquad (26)$$

From this variational problem Schrödinger derived the wave equation for the hydrogen atom:

$$\nabla^2\Psi + \frac{2m}{K^2}\left(E + \frac{e^2}{r}\right)\Psi = 0 \tag{27}$$

This derivation presented in Schrödinger's first 1926 paper is completely meaningless, since Ψ was an undefined function and (26) is not a valid variational principle in classical physics[5]. Its only "justification" is that the wave equation leads to the correct energy levels of the hydrogen atom, carefully derived in the rest of the same paper (Schrödinger, 1926a, p. 362-374; 1929, pp. 2-10).

9. THE DERIVATION IN SCHRÖDINGER'S SECOND PAPER

Schrödinger himself was not satisfied with this "derivation", and presented a very different one in his second 1926 paper (Schrödinger, 1926b; Schrödinger, 1929, pp. 13-40), where he presented for the first time the time-independent wave equation (*m*=1)

$$\Delta\Psi + \frac{8\pi^2 \mathrm{m}}{\mathrm{h}^2}(\mathrm{E} - \mathrm{U})\Psi = 0 \tag{28}$$

We present here a reconstruction of his derivation, stressing only its main points.

Schrödinger's fundamental assumption was the general (classical) wave equation in this form (Schrödinger, 1926b, p. 510; Schrödinger, 1929, p. 27)[6]:

[5] "The first published derivation [...] was not only curiously formal, but straighforwardly cryptical. On the whole this derivation appears badly justified, its sole foundation lying in its result [...]" (Kragh, 1982, p. 158).

[6] Instead of the usual symbol for the Laplacian operator used here, Schrödinger wrote *div grad*.

$$\nabla^2\psi - \frac{1}{u^2}\ddot{\psi} = 0 \tag{29}$$

Schrödinger had already obtained (Schrödinger, 1926b, pp. 494-498; Schrödinger, 1929, pp. 16-20) the equation of the speed of the wave (m=1):

$$u = \frac{ds}{dt} = \frac{E}{\sqrt{2(E-V)}} = \frac{h\nu}{\sqrt{2(h\nu - V)}} \tag{30}$$

Now, assuming that the wave function Ψ has the same form it usually has in classical physics (Schrödinger, 1926b, p. 510; Schrödinger, 1929, p. 27):

$$\psi = f(q_k).e^{2\pi i\nu t} \tag{31}$$

we derive:

$$\ddot{\psi} = \frac{\partial^2\psi}{\partial t^2} = (2\pi i\nu)^2.f(q_k).e^{2\pi i\nu t} = -4\pi^2\nu^2\psi \tag{32}$$

By replacing (32) and (30) in (29) one obtains at once Schrödinger's wave equation (m=1) :

$$\nabla^2\psi + \frac{8\pi^2}{h^2}(h\nu - V)\psi = 0 \tag{33}$$

Since $E=h\nu$, it may also be written as (Schrödinger, 1926b, p. 510; Schrödinger, 1929, p. 27):

$$\nabla^2\psi + \frac{8\pi^2}{h^2}(E - V)\psi = 0 \tag{34}$$

Therefore, in this derivation presented in Schrödinger's second 1926 paper, the only non-classical assumptions are $E=h\nu$ and the formula for the speed u of the waves associated to the electron (30) that corresponds to the classical limit of De Broglie's relation $u=E/p$.

However, Schrödinger did not take the formula for *u* from De Broglie's work. He presented his own, original and highly abstract derivation of this relation, using the analogy between the principles of Huygens and Hamilton (Schrödinger, 1926b, pp. 494-498; Schrödinger, 1929, pp. 16-20). He referred to De Broglie's work in this paper, but stressed that his own results were obtained in a more general way, and independently of the theory of relativity. After presenting his own derivation, he remarked:

> We find here again a theorem for the 'phase waves' of the electron, which Mr. de Broglie had derived, with essential reference to the relativity theory, in those fine researches to which I owe the inspiration for this work. We see that the theorem in question is of wide generality, and does not arise solely from relativity theory, but is valid for every conservative system of ordinary mechanics. (Schrödinger, 1926b, p. 498; Schrödinger, 1929, p. 20)

So, Schrödinger *could* have used De Broglie's work to derive the wave equation in the non-relativistic case, *but he did not do that*. He used a formula for the speed of the wave associated to the electron that is equivalent to De Broglie's, but he provided a new derivation of this relation – seemingly because he wanted to prove that it was valid independently of relativistic considerations.

10. SCHRÖDINGER'S AND DE BROGLIE'S THEORIES

We might justify Schrödinger's careful avoidance of grounding his own work upon De Broglie's theory in the following way. He had first attempted to use the relativistic approach, and it failed; besides that, the wave equation that did lead to correct results is not the classical limit of the relativistic wave equation. Therefore, it was appropriate to present a derivation completely independent of De Broglie's relativistic theory, to ensure that his own theory was well grounded. Probably this was part of his motivation. However, there are other relevant issues, since there are several deep differences between the two theories.

De Broglie's work was grounded upon special relativity, and its main equations could only be deduced for electrons in uniform motion, since he took as his starting point the description of the pulsation of the extended electron in its rest frame. Its extension to accelerated motion required a new way of thinking, and a different justification. De Broglie did attempt to provide a basis for his theory in the case of accelerated motion using the analogy between the principles of Maupertuis and Fermat – that are valid only for single particles and wave rays. This led him to a general relation between the wave properties and the mechanical properties of the electron moving in an electromagnetic field. However, De Broglie's electron still described a definite trajectory and the associated wave could be described by wave rays – an approximation which is adequate in describing phenomena corresponding to geometrical optics, but completely inadequate for the analysis of diffraction and interference. He successfully applied his ideas to the hydrogen atom, but he was unable to study the harmonic oscillator and other simple systems. De Broglie's concept of the electron could not be adequately applied when the dimensions of the system was comparable to the wavelength (as in the case of the atom). It was also difficult to perceive how his theory could be applied to a system with many particles. Besides that, his theory would be meaningless for a rigid rotator, for instance.[7]

Schrödinger's approach was much more general than De Broglie's. His derivations did not depend on the theory of relativity and therefore he could directly address the case of accelerated and rotating systems. Besides that, in the derivation presented in his second 1926 paper (Schrödinger, 1926b, 490-491; 1928, p. 14) he presented his own theory in a very general

[7] The discussion of the quantum rigid rotator was very important in the theory of specific heat of poliatomic gases. This problem was addressed by Schrödinger in his second 1926 paper.

way, using general coordinates (therefore opening the possibility of applying it to rotation and other types of motion) and discussing a general conservative system (that is, he did not restrict his treatment to a single particle). His general and abstract way of approaching the problem allowed him to apply the wave equation to any physical system.

In his second paper, Schrödinger used the analogy between the principles of Huygens and Hamilton (not Fermat and Maupertuis, as De Broglie did), and that step ensured that the relation could be carried to cases where the dimensions of the system were comparable to the wavelength.

For those reasons, only Schrödinger's theory could be directly applied to waves associated to one or more electrons in three dimensions (atomic systems), to the harmonic oscillator and to rotating solids, as he did (Schrödinger, 1926b).

11. CONCLUSIONS

Schrödinger's wave equation can be derived from De Broglie's results, in the classical limit. However, Schrödinger's theory is not an application or development of De Broglie's theory.

Schrödinger's theory can be applied to cases where De Broglie's theory cannot be applied, such as accelerated motion and rotation, and for bound particles where the wavelength is comparable to the dimensions of the region containing the electron. Even in the case of a free particle they are also incompatible: De Broglie's *relativistic* wavelength was confirmed by diffraction experiments, and Schrödinger's classical wavelength is simply wrong, for high speeds.

The heuristic value of Schrödinger's wave equation is another very important distinction between the two theories, since de Broglie's theory only led to a single new prediction: the wave behaviour of electrons in diffraction experiments. There are also other differences between the two approaches that cannot be described here.

Although De Broglie's theory was the starting point of Schrödinger's work and had a very important heuristic role in this respect, the two theories are different, independent and incompatible.

ACKNOWLEDGEMENTS

The author is grateful to the Brazilian National Council for Scientific and Technological Development (CNPq) and to the São Paulo State Research Foundation (FAPESP) for supporting this research.

BIBLIOGRAPHIC REFERENCES

BORK, Alfred M. Maxwell and the vector potential. *Isis*, **58**: 210-222, 1967.

BROGLIE, Louis de. Ondes et quanta. *Comptes Rendus de l'Académie des Sciences de Paris*, **177**: 507-510, 1923 (a).

BROGLIE, Louis de. Quanta de lumière, diffraction et interférences. *Comptes Rendus de l'Académie des Sciences de Paris*, **177**: 548-550, 1923 (b).

BROGLIE, Louis de. Les quanta, la théorie cinétique des gaz et le principe de Fermat. *Comptes Rendus de l'Académie des Sciences de Paris*, **177**: 630-632, 1923 (c).

BROGLIE, Louis de. Waves and quanta. *Nature*, **112**: 540, 1923 (d).

BROGLIE, Louis de. A tentative theory of light quanta. *Philosophical Magazine*, **47**: 446-458, 1924 (a).

BROGLIE, Louis de. Sur la définition générale de la correspondance entre onde et mouvement. *Comptes Rendus de l'Académie des Sciences de Paris*, **179**: 39-40, 1924 (b).

BROGLIE, Louis de. Sur un théorème de M. Bohr. *Comptes Rendus de l'Académie des Sciences de Paris*, **179**: 676-677, 1924 (c).

BROGLIE, Louis de. *Thèses. Recherches sur la théorie des quanta.* Paris: Masson, 1924 (d).

BROGLIE, Louis de. Recherches sur la théorie des quanta. *Annales de Physique*, series 10, **3**: 22-128, 1925.

BROWN, Harvey Robert; MARTINS, Roberto de Andrade. De Broglie's relativistic phase waves and waves groups. *American Journal of Physics*, **52**: 1130-1140, 1984.

DARRIGOL, Olivier. Strangeness and soundness in Louis de Broglie's early works. *Physis: Rivista Internazionale di Storia della Scienza*, **30**: 303-372, 1993.

HANLE, Paul A. Erwin Schrödinger's reaction to Louis de Broglie's thesis on the quantum theory. *Isis*, **68**: 606-609, 1977.

HANLE, Paul A. The Schrödinger-Einstein correspondence and the sources of wave mechanics. *American Journal of Physics*, **47**: 644-648, 1979.

HEITLER, Walter. Erwin Schrödinger, 1887-1961. *Biographical Memoirs of Fellows of the Royal Society*, **7**: 221-228, 1961.

JAMMER, Max. *The conceptual development of quantum mechanics.* New York: MacGraw-Hill, 1966.

KRAGH, Helge. Erwin Schrödinger and the wave equation: the crucial phase. *Centaurus*, **26**, 154-197, 1982.

KRAGH, Helge. Equation with the many fathers. The Klein-Gordon equation in 1926. *American Journal of Physics*, **52**, 1024-1033, 1984.

MARTINS, Roberto de Andrade. De Louis de Broglie a Erwin Schrödinger: uma comparação. Pp. 393-409, in: FREIRE JR, Olival; PESSOA JR., Osvaldo; BROMBERG, Joan Lisa (orgs). *Teoria quântica: estudos históricos e implicações culturais*. Campina Grande: EDUEPB; São Paulo: Livraria da Física, 2010.

MARTINS, Roberto de Andrade; ROSA, Pedro Sérgio. *História da teoria quântica: a dualidade onda-partícula, de Einstein a De Broglie*. São Paulo: Livraria da Física, 2014.

MEHRA, Jagdish; RECHENBERG, Helmut. *The historical development of quantum theory*. 5 vols. New York: Springer, 1982-1987.

MOORE, W. J. *Schrödinger: life and thought*. Cambridge: Cambridge University, 1989.

RAMAN, V. V.; FORMAN, Paul. Why was it Schrödinger who developed de Broglie's ideas? *Historical Studies in the Physical Sciences*, **1**: 291-314, 1969.

REGT, Henk W. de. Erwin Schrödinger, Anschaulichkeit, and quantum theory. *Studies in History and Philosophy of Modern Physics*, **28**: 461-481, 1997.

SCHRÖDINGER, Erwin. Quantisierung als Eigenwertproblem (Erste Mitteilung). *Annalen der Physik*, series 4, **79**: 361-376, 1926 (a).

SCHRÖDINGER, Erwin. Quantisierung als Eigenwertproblem (Zweite Mitteilung). *Annalen der Physik*, series 4, **79**: 489-527, 1926 (b).

SCHRÖDINGER, Erwin. *Collected papers on wave mechanics*. London & Glasgow: Blackie & Son, 1929.

WHEATON, Bruce R. *The tiger and the shark. Empirical roots of wave-particle dualism*. London: Cambridge University, 1983.

Scientiarum Historia et Theoria Studia, volume 3

Roberto de Andrade Martins

Studies in History and Philosophy of Science II

Extrema: Quamcumque Editum, 2021

Summary

Foreword 1

A priori components of science: Lavoisier and the law of conservation of mass in chemical reactions 5

Jevons and the role of analogies in empirical research 73

Wave mechanics, from Louis de Broglie to Schrödinger: a comparison 105

The cultural relevance of astronomy in classical Antiquity 125

The transformation of astronomical culture in the seventeenth century 175

Paperback edition: ISBN 978-65-996890-3-1
Kindle edition: ISBN 978-65-996890-4-8

Available at:
https://www.amazon.com/dp/6599689035