Philosophy in the physics laboratory: measurement theory versus operationalism

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Abstract. This paper presents Helmholtz' approach to measurement theory, and discusses the possible uses and implications of his views in physics education. This approach opposes the "black box" attitude towards measurement (operationalism). It takes into account the *a priori* conditions that should be imposed upon measurement procedures to obtain results that conform to some basic mathematical properties of physical magnitudes. Instead of the discredited but still popular empiricist view that the scientist should observe nature without any preconceived ideas, Helmholtz' theory of physical magnitudes shows that in basic measurement procedures it is necessary to introduce theoretical considerations. At the same time, this does not introduce a vicious circle in experimental testing.

Introduction

Among physicist, the phrase "measurement theory" is usually associated to operationalism, inductivism, and the study of random errors. Measuring instruments and, in general, scientific instruments, are regarded as "black boxes" that produce readings when applied to physical systems. The measuring instrument "defines" a physical magnitude. Therefore, when two different instruments (or methods) are applied to the same object, they can produce different results, because they are not measuring the same property.

Every physics student learns that measurement is one of the most important activities in the physics laboratory. In the educational laboratory, measuring apparatuses are used to obtain data; they are instruments, and therefore they are never the subject of study in the physics lab. Textbooks about measurement and laboratory techniques usually devote just a few paragraphs to systematic errors. Only random errors deserve a detailed study.

In the context of the usual approach, it is not clear what meaning can be ascribed to questions such as this: Is the measuring instrument working properly and measuring what it was intended to measure? If a physical magnitude is *defined* by its measuring method, it is impossible to criticise or improve the method. If distances are measured (and operationally defined) by the instruments that measure distance, it is meaningless to ask whether the ruler measures distances *correctly* or not.

Measurement devices are not, however, blindly made instruments. Also, it is not necessary to have a blind faith in their performance. It is possible to analyse and test measurement instruments; and there is a general theory of measurement instruments and a general theory of systematic errors that can be useful in real-life laboratory.

However, the required background is not operationalism, but a completely different approach. Before the advent of operationalism in the early 20th century, there existed already a deeper analysis of measurement theory provided by Hermann von Helmholtz (1887). This approach takes into account the *a priori* conditions that should be imposed upon measurement procedures in order to obtain results that conform to some basic properties of physical magnitudes. Helmholtz' approach was followed and developed by

Norman Campbell (1920). In more recent times, one can find an excellent textbook on this subject written by a philosopher: Brian Ellis (1968).

This paper will present a short account of Helmholtz' approach to measurement theory, and will discuss the possible uses and implications of this approach in physics education.

Helmholtz' contribution

Hermann von Helmholtz' famous analysis of measurement is contained in his 1887 essay *Zählen und Messen erkenntnis-theoretisch betrachet*. That writing was part of Helmholtz' discussion of Kant's views on the relations between science and experience. In some former papers, Helmholtz had argued that the axioms of geometry are not propositions given *a priori*, but rather that they are to be established or refuted through experience. In a similar vein, in his 1887 essay he discussed the axioms of arithmetic and attempted to unravel their empirical content.

The first part of Helmholtz' essay is dedicated to the discussion of number, arithmetic and counting. In the second part of his essay, he discusses measurement.

Helmholtz presented the following traditional axioms of arithmetic[1]:

Axiom I. If each of two magnitudes is equal to a third, they are equal to each other.
Axiom II. Associative law of addition: (a+b)+c=a+(b+c)
Axiom III. Commutative law of addition: a+b=b+a
Axiom IV. Equals added to equals give equals.
Axiom V. Equals added to unequals give unequals.

A large part of Helmholtz' paper was devoted to the discussion of those axioms and the mathematical concept of number. He showed, for instance – following Hermann and Robert Grassmann – that it is possible to reduce axioms II and III to another one, namely, (a+b)+1=a+(b+1). This part of his essay will not be discussed here. Let it be said, however, that his approach to the foundation of arithmetic was regarded as naïve by most mathematicians of that time, and that this field of investigation soon gained rigour and clearness. Dedekind, Cantor and Frege despised Helmholtz' approach that mixed up mathematics with empirical issues (Darrigol, 2003, pp. 518, 557-561).

In the second part of his work, Helmholtz introduced the concept of *magnitude*: "Objects or attributes of objects, which, when compared with similar ones, permit the distinction of greater, equal, or smaller, we call magnitudes" (Helmholtz, 1930, p. 17). In some cases (but not always) it is possible to ascribe numbers to magnitudes. "The procedure whereby we find the denominate number we call the measuring of magnitudes" (Helmholtz, 1930, p. 17).

Under what conditions can numbers and their operations be applied to the relations of real objects and their magnitudes? He reduced this problem to two simpler ones: the empirical meaning of *equality* and of *addition* of magnitudes.

> 1. What is the objective meaning of declaring two objects in a certain relation equal?
> 2. What character must the physical combination of two objets have in order that we may consider comparable attributes of the same as united additively and these

[1] Helmholtz proposed an additional axiom (Axiom VI): If two numbers are different, one of them must be higher than the other.

> attributes accordingly as magnitudes which can be designated by denominate[2] numbers? (Helmholtz, 1930, p. 4)

The comparison of two magnitudes

The first point Helmholtz elucidated was the meaning of equality (or inequality) as applied to the attributes of objects. If we ascribe to this concept (physical equality) the same properties that he had already described in the case of numbers, then the new concept must obey the following properties:

(P1) if a=b then b=a (symmetry)

(P2) if a=c and b=c then a=b (transitivity)

Now, assume we have a method for comparing some attribute of different objects (for instance, their size or weight). If we compare objects A and B and conclude that some of their attributes (size, weight, etc.) are equal, we should expect that the comparison of B and A will also lead to the same conclusion, by (P1).

Suppose, for instance, that we compare the weights of two bodies using the kind of balance that was employed in the 19th century. If the two bodies A and B are in equilibrium, we conclude that their weights are equal. If we exchange the positions of A and B, the balance must remain in equilibrium. If this does not occur, the instrument is inadequate (Helmholtz, 1930, pp. 19-20).

Notice that the comparison of weight, in this case, *precedes* the procedure for ascribing weights to the two bodies. That is, the comparison between the weights of A and B is not a comparison between two numbers, since we can ascertain the equality between their weights without knowing either weight.

Helmholtz pointed out that the second property (P2) does also have an empirical meaning (Helmholtz, 1930, p. 20). If we use a two-pan balance to compare the weights of A and C and we observe that they equilibrate each other; and if B and C do also equilibrate each other; we should expect that A and B will also equilibrate each other. If that does not occur, the instrument is inadequate.

Therefore, properties (P1) and (P2) can be used to test comparison instruments. A "correct" instrument should pass tests grounded upon properties (P1) and (P2). Of course, it is not possible to prove that the instrument is correct, but it may be possible to find out that it is incorrect. Those properties determine what physical relations we can recognise as relations of equality (Helmholtz, 1930, p. 22).

When a given comparison procedure is proposed and tested, the results can conform to the expected results (taking into account the mathematical properties of equality) or they may fail to conform to the predicted results. In the second case, the comparison procedure is rejected. However, in principle, the *mathematical properties of equality* might as well be rejected. Why are they never put to test? Because we have a preconceived (*a priori*) idea of mathematical equality which does not come from experience and that will not be invalidated by any observation.

Notice that Helmholtz introduced *comparison methods* in a way that precedes and is independent of ascertaining the *value* of the corresponding magnitudes. In our time, if we ask someone who is not familiar with measurement theory how could he/she determine whether two objects have the same length or weight, the most likely answer would be that we should measure both objects and compare their measurements. However, there are procedures which are prior to measurement proper and that allow us to *compare* objects as regards some specific quantities. We can see whether two persons are equally tall without

[2] "Denominate" numbers are those accompanied by units.

measuring their sizes, by putting them side by side. We can test whether two bodies have the same weight by putting them on the two pans of an equal-arm balance, and checking whether the balance remains in equilibrium or not. This is a fundamental idea, which should be stressed in elementary physics courses.

Physical addition of two magnitudes

Mere comparison of two magnitudes can show whether they are equal or unequal, but does not provide measurements (numbers) to those magnitudes. If we are to describe those magnitudes by numbers, and if those numbers are to be used in arithmetical operations, they must obey some conditions. As the basis of all arithmetical operations is addition, Helmholtz first analysed "under what conditions we can express a physical combination of magnitudes of the same denomination as an addition" (Helmholtz, 1930, p. 22).

We add the lengths of two bodies by placing some of their extremities in contact with each other and by putting the contact point and their other ends in a straight line (Helmholtz, 1930, p. 23). In the case of weights, the physical combination of A and B is a mere juxtaposition of the two objects (Helmholtz, 1930, p. 22). It does not matter if the two objects are put side by side, or one on the top of the other, or slightly separated. If we put the two objects on one pan of a two-pan balance, they will always be equilibrated by the same object C on the other pan. Therefore, the physical combination of two objects as regards their weights does obey a law similar to the commutative law of addition (P3): $a+b=b+a$.

This is, of course, an empirical finding. If A on the top of B equilibrated C, but B on the top of A did not equilibrate C, then the physical combination of A and B as regards their weight would not be a mere juxtaposition of the two objects. That is: a given rule of physical combination of magnitudes can be tested by comparing it to the properties of numbers. If the rule does not pass the test, it is an inadequate rule and should be rejected.

For other properties, the physical combination that produces the addition of their magnitudes is different.

> We add resistances when we unite the wires one after the other so that the electricity conducted through them must flow through each successively. We add conductivity of the wires when we put the wires side by side and unite all their beginnings and also all their ends. (Helmholtz, 1930, p. 25)

It is possible to apply several tests, using other laws of arithmetical addition, such as the axiom "equals added to equals give equals" (P4): if $a=b$ and $c=d$ then $a+c=b+d$

That is, if bodies A and B are equal as regards their weight, and C and D do also equilibrate, then we should expect that the combination of A+C will equilibrate B+D. Also, if A+C equilibrates another body E, then we should expect that A+D, B+C and B+D should also equilibrate E. If that does not occur, then either the comparison rule (the balance) or the combination rule (the juxtaposition of objects) is inadequate. If all those properties are obeyed, then the combination rule can be (temporarily) accepted as an adequate physical counterpart of arithmetical addition.

> A physical method of combining magnitudes of like kind can then be regarded as addition, if the result of the combination, compared as magnitudes of the same kind, is not changed, either by the interchange of single elements or by the exchange of members of the combinations with equal magnitudes of the same kind. (Helmholtz, 1930, p. 24)

It is not possible to apply this analysis to some other magnitudes, such as temperature and density, because when we join two objects with temperatures (or densities) A and B, the temperature (density) of the system does not become C=A+B.

Notice that we can physically add many (but not all) physical magnitudes. It is possible to add the volumes of two non-reacting liquids by putting them in the same vessel; it is possible to add the resistances of two pieces of wire joining them in series; it is possible to add potential differences in the same way. It is possible to add currents by joining parallel wires. It is possible to add the intensities of two incoherent radiation sources by their simultaneous action upon the same surface. Using those properties, it is possible to test the linearity of instruments that measure volume, resistance, potential difference, electric current, radiation intensity, etc.

Notice that there is an empirical component and an *a priori* component in the process of physical addition. According to our *concept* of physical magnitudes, we have the need of associating numbers to magnitudes and of performing arithmetical operations with them. This would be pointless if there was no relationship between arithmetical operations and physical properties. It is meaningless, for instance, to perform arithmetical operations with telephone numbers or library classification numbers of books. It is meaningful, however, to perform arithmetical operations with length measurements.

In the case of some physical magnitudes, it is possible to find physical operations that obey the same properties as arithmetical addition. For instance: if we join two objects together, the weight of the new (compound) object is the sum of the masses of the two objects. This property was not derived from measurement: it is an *a priori* requirement imposed upon measurement. Any balance (even the most expensive model of a beautifully illustrated catalogue) that refused to obey this law of addition of weight should be rejected as inexact. However, it is an empirical matter to check whether this addition rule does really obey the same rules as the arithmetical addition.

If we know how to compare two magnitudes of the same kind without measuring them, and if we know how to add two magnitudes of the same kind by joining two physical systems in a specific way, then we can create a measurement procedure for that magnitude. The typical instance is again provided by the old equal-arms balance together with a set of standard weights. It is possible to combine the standard weights and it is possible to compare their combination to any given body. In that way, it is possible to ascribe numbers (measurements) to those bodies.

Precedents of Helmholtz' work

Olivier Darrigol published a detailed analysis of the authors who probably influenced Helmholtz' theory of measurement, and the influence exerted by Helmholtz' ideas (Darrigol, 2003).

Helmholtz was inspired by mathematicians (such as the Grassmann brothers, Hermann and Robert, and Paul Du Bois-Reymond) and by physiologists / psychologists who discussed the possibility of measuring psychological quantities (Darrigol, 2003, pp. 520-541). He was probably influenced by Ernst Mach and James Clerk, too. There are moreover several similarities between Maxwell´s previous discussion of measurement of electric charge and temperature and Helmholtz' ideas (Darrigol, 2003, pp. 541, 548).

In his *Theory of heat* (1871) Maxwell discussed the measurement of temperature and emphasized that the equality between the temperatures of two bodies is a concept more fundamental than the value of their temperatures, and that it can be ascertained by putting the two bodies in contact and checking whether there was any heat flow from one to the other. However, if this test is to be used for ascertaining equality of temperature, it must obey a testable law: if the temperatures of A and B are equal to the temperature of C, then the temperatures of A and B should also be equal. If a piece of iron is plunge in a vessel of water and it is noticed that they are in thermal equilibrium; and if the same piece of iron, being transferred to a vessel full of oil, is observed to be in thermal equilibrium with the oil; than it is possible to predict (and to check) that the water and the oil will be in thermal equilibrium (Darrigol, 2003, p. 542). Nowadays this law is sometimes called “the zeroth law of thermodynamics”. It is seldom ascribed to Maxwell, and its relation to the general theory of measurement is never mentioned.

Maxwell also remarked that it is impossible to add two bodies at temperatures P and Q producing a third body at the temperature P+Q. If there were such a procedure, then it would be possible to measure temperatures in the same way we measure mass or length.

Although there were some precedents to the ideas presented by Helmholtz, as was shown above, it seems that his essay was the first systematic discussion of the measurability of physical properties (Darrigol, 2003, p. 516).

Campbell’s contribution

Helmholtz’ analysis is very useful to check basic measurement procedures in order to detect systematic errors. However, his treatment had several shortcomings. In his paper, Helmholtz did not discuss random errors. Also, he did not analyse other kinds of measurement, that apply to magnitudes for which there is not a procedure of physical addition (such as density). Norman Campbell, in his 1920 book *Physics, the elements*, presented a more complete and detailed (and, in some senses, more satisfactory) account of measurement. In his book, Campbell never referred to Helmholtz; however, directly or indirectly, he certainly suffered his influence.

Campbell introduced the name “fundamental measurement” to characterize the direct measurement procedures (those that do not depend on the measurement of other quantities), which are grounded upon comparison and physical addition (Campbell, 1957, pp. 277-278) – that is, those that were studied by Helmholtz. Of course, Helmholtz knew (and stated) that some quantities (such as temperature and density) cannot be measured in the same way as weight or length, because there is no known way of producing the physical addition of two systems as regards those properties. However, Helmholtz did not give *names* do the different cases, nor did he elucidate other forms of measurement. Campbell distinguished several cases, and gave the name *derived measurement* to the measurement of quantities such as density.

Instead of a relation of *equality* between two quantities, Campbell used another comparison relation, that of *greater than*, or *smaller than*. The physical order relation must be a transitive and asymmetrical relation, as the corresponding mathematical relation – and it is possible to *test* whether a given physical comparison method does obey or does not obey those properties (Campbell, 1957, pp. 270-274).

Helmholtz did not discuss random errors and the difficulty they introduce in fundamental measurement. It may happen that some comparison method shows that A and B are equal (relative to some magnitude), and that B and C are also equal, but A and C are different. In order do deal with this situation it is necessary to introduce the concept of errors of measurement (or uncertainty). The comparison between two objects can only lead to the result that their difference is *smaller than the error of measurement*, but cannot establish that they are *equal.* Campbell developed the analysis of measurement taking into account the existence of errors (Campbell, 1957, chapter 16), while Helmholtz did not.

Although there are relevant differences between Campbell's and Helmholtz' approaches, both authors emphasised that the fundamental measurement of physical magnitudes involves operations that must obey a set of laws isomorphic to those of arithmetic[3]; and that it is possible to check whether a given operation (or instrument) does obey such a law, so that systematic errors (or "methodical errors", according to Campbell) can be eventually found.

If we take into account random errors, Helmholtz rules must be corrected. Suppose we measure the weight of two objects using a balance, and that their measurements are repeated several times, and their weights are $A \pm a$ and $B \pm b$. When both objects are put together on the balance pan, we should obtain a weight C compatible with $(A+B) \pm (a^2+b^2)^{1/2}$. Otherwise, we should conclude that the balance has a systematic error, since it is unable to add.

In the same way, suppose we measure the thickness of two plates using a micrometer, and obtain the values $A \pm a$ and $B \pm b$. When both plates are put together and measured with the same micrometer, we should obtain a thickness $C \pm c$ compatible with $(A+B) \pm (a^2+b^2)^{1/2}$.

It would be possible to present many other relevant features of Helmholtz' and Campbell's theories of measurement. Those that were shown here are sufficient, however, to exhibit the central ideas of this approach to measurement theory.

The educational use of measurement theory

The discussion of these and other fundamental issues concerning the theory of physical magnitudes and their measurement can be introduced in our teaching practice in order to bring a new light to experimental science. In contrast to the discredited but still popular empiricist view that "the scientist should observe nature without any preconceived ideas", the theory of measurement shows that it is necessary to introduce theoretical considerations in basic measurement procedures. In addition to random errors there are other kinds of errors, and there is a *theory* about the way instruments should behave that allow us to test them and to impose some conditions before we adopt a measuring instrument.

It is necessary to remark that the idea that there is a theory underlying our measuring instruments is indeed generally recognized nowadays. This truism is part of the post-modern philosophy of science and is usually presented as an argument against the objectivity of quantitative science. If theories are needed to build measurement instruments, then the measurements obtained with the use of those instruments are influenced by theory, and cannot be used to *test* a theory, because this would entail a vicious circle: the theory should be accepted because it was confirmed by some measurements, and the measurements should be accepted because they are grounded upon the theory.

There is no vicious circle, however, because the theory under the measurement apparatus is not the same theory that is assessed with the use of that apparatus. There are

[3] As a matter of fact, there is not an *isomorphism* between the physical quantities of the objects and arithmetic, but a weaker kind of *morphism*. Those distinctions will not be introduced here, however. They are discussed in several works in the mathematical tradition of measurement theory, beginning with Patrick Suppes (1951). A nice historical analysis of measurement theory which discusses those issues can be found in José Díez (1997).

testable physical laws underlying the functioning of the measurement apparatus, but those physical laws are the laws of measurement, analogous to the laws of arithmetic, such as those presented by Helmholtz. Taking into account the theory of measurement, the devices can be tested (i.e., checked for systematic errors) before being applied to the test of scientific theories.

An acquaintance with measurement theory can provide a more adequate view of the nature of experimental science, and it can also provide an effective help in discussing and searching for systematic errors in the physics laboratory.

Both Helmholtz and Campbell believed that the theory of measurement could be useful in physics courses. Helmholtz reproduced his analysis of measurement in two of his textbooks, and Campbell explicitly recommended the inclusion of a general discussion of measurement in introductory physics courses (Darrigol, 2003, pp. 554-555, 569-570). However, neither Helmholtz nor Campbell were successful in disseminating those ideas among physicists and physics teachers.

The theory of measurement developed by Helmholtz, Campbell and other authors was not integrated into physics textbooks. It was discussed by philosophers and by psychologists and incorporated in their works. In the context of physics teaching, either measurement theory is completely ignored, or it appears under the form of *operationalism*.

The operational approach in physics teaching

The operational approach to measurement emerged in the early 20th century from the work of physicists, and had a strong influence upon other fields. The main representative of this approach was Percy W. Bridgman, who published a book (*The logic of modern physics*) and several papers on his theory.

According to operationalism, the measuring procedure *defines* a scientific magnitude. If we do not know how to measure something, it should not be regarded as a scientific magnitude. If we know how to measure it, then we know all that can be known about that magnitude. The measurement procedure, being a definition, is a *convention*. We can choose whatever procedure we want.

Of course, there *was* a positive characteristic of operationalism: it emphasized the importance and utility of describing how is it possible to measure a given magnitude – when that is possible. There are many relevant physical magnitudes (such as the vector potential of electromagnetism) that cannot be measured at all. If we accepted the operational point of view, we should reject them. Now, it would be very useful to have some procedure to measure those quantities; however, even though we cannot measure them, they are useful in physics.

Even in the cases in which we *can* measure a magnitude, the measurement procedure is not a *definition* of that quantity, and it is not arbitrary. If the instrument *defined* the magnitude, it would be impossible to say that the instrument is *wrong*. That is a relevant point that is stressed in the present paper. There exists a *measurement theory*, and that theory can tell use whether a given measurement procedure is acceptable or not (at least in some cases).

Up to the decade of 1950, operationalism was popular among scientists and philosophers. During the second half of the 20th century, however, it suffered heavy attacks from philosophers and was completely discredited.

Although rejected by philosophers, operationalism retained its appeal to physicists. In the last decades of the 20th century operationalism still appeared explicitly in educational articles as the accepted theory of measurement. In a paper describing some novelties introduced in the physics laboratory at the University of California, Berkeley, in 1979, the authors described that they expected the students to learn some general skills, “those which practicing scientists commonly use, but which most students do not possess”. The “general skills” were:

> (i) **Being able to use operational definitions to relate symbolic concepts to observable quantities**. This skill subsumes the ability to estimate or measure important physical quantities at various levels of precision. (ii) Being able to estimate the errors of quantities obtained from measurements. This skill involves applying habitually some qualitative or semiquantitative statistical notions, without any resort to excessive mathematical formalism. (iii) Knowing and applying some generally useful measuring techniques for improving reliability and precision, for example, such techniques include making repeated measurements, using independent measurement methods, or applying comparison or “null” methods. (Reif & St. John, 1979, p. 950; my emphasis).

It is possible to find more recent papers published in physical journals that still regard operationalism as a viable theory of measurement (Delaney, 1999), although physicists have been told by Mario Bunge (and other authors) that operationalism is scientifically and philosophically inadequate (Bunge, 1967).

Several generations of physicists and physics teachers, all over the world, have studied Resnick and Halliday’s physics textbook (Resnick & Halliday, 1966). Both in the first and in the following editions, the first chapter of that treatise deals with measurement. In the oldest edition, the approach used by the authors is operationalism:

> For the purposes of physics the basic quantities must be defined clearly and precisely. One view is that the definition of a physical quantity has been given when the procedures for measuring that quantity have been given. This is the *operational point of view* because the definition is, at root, a set of laboratory operations leading to a number with a unit. The operations may include mathematical calculations.
>
> […]
>
> Examples of quantities usually viewed as fundamental are length and time. Their operational definitions involve two steps: first, the choice of a *standard*, and second, the establishment of procedures for comparing the standard to the quantity to be measured so that a number and a unit are determined as the measure of that quantity. (Resnick & Halliday, 1966, vol. 1, p. 2)

In more recent editions of this textbook, no explicit reference is made to the *operational point of view*, but the shallow concept of measurement presented in that manual is the same: the most important step in measurement is choosing a standard, and then, in some way that is not elucidated, the measured system is compared to the standard and there results some number. Notice that this was the way physics textbooks introduced measurement before Helmholtz; and this is the way measurement is still presented by most physics teachers (Darrigol, 2003, p. 565).

The common attitude transmitted by the best physics laboratory textbooks is to ignore altogether the existence of measurement theory, paying attention mostly to statistics, and mentioning systematic errors *en passant*.

> Random errors may be estimated by statistical methods, which are discussed in the next two chapters. Systematic errors do not lend themselves to any such clear-cut treatment. Your safest course is to regard them as effects to be discovered and eliminated. There is no general rule for doing this. It is a case of thinking about the particular method of doing an experiment and of always being suspicious of the apparatus. We shall try to point out common sources of systematic error in this book, but in this matter there is no substitute for experience. (Squires, 1991, p. 11)

As a matter of fact, *there are general rules* that can be used to search for systematic errors. It is true that no rule will detect *all* systematic errors, but *many* systematic errors can be found if one follows some simple rules of measurement theory.

A recent multi-author paper that appeared recently in *The Physics Teacher* addresses the teaching of measurement (Allie *et al.*, 2003). The main concern of the paper is how to teach students to deal with uncertainty in the introductory physics laboratory. Although the article does mention systematic errors, the approach of the authors is probabilistic and they do not address measurement theory.

Nowadays, measurement theory is ignored in scientific education. It is not mentioned in *Science for all Americans / Project 2061* of the American Association for the Advancement of Science. According to the guidelines of that project, it is expected that everyone should acquire the ability to "use appropriate instrument to make direct measurements of length, volume, weight, time interval, and temperature [...]" and also to "take readings from standard meter displays, both analog and digital, and make prescribed settings on dials, meters, and switches" (Rutherford, 1990, pp. 191-192). However, *understanding* the instruments and the principles underlying measurement is not included in the aims of that Project.

In the context of science education, few warnings have been published regarding the problems of operationalism. One nice exception can be found in a paper by Jorge Paruelo, where the author calls the attention to contradictions that can be found in physics textbooks which adopt the operational point of view:

> The contradiction is a consequence of not distinguishing the operationalization of a magnitude – that is, defining an operative mechanism to detect the presence of the magnitude, or to measure its quantity – and operationalism – that is, defining the magnitude by the aforementioned operation. A correct epistemological analysis of this subject will allow the development of a new route for teaching what a physical magnitude is and how it is detected. (Paruelo, 2003, p. 331)

I hope that the present paper will help calling the attention of physicists to measurement theory, and will assist in improving the teaching of experimental physics. I do not claim that the ideas presented here can solve all the educational problems concerning the science laboratory. I do claim, however, that the subject of measurement is almost exclusively viewed by teachers and students as the blind use of measurement instruments and later statistical manipulation of data and that this is an inadequate view. Of course, even on that viewpoint there are educational problems – for instance, students have difficulty in understanding the need for repeating measurements and in dealing and interpreting uncertainties, and it is useful to investigate how those difficulties can be circumvented (Rollnick *et al.*, 2002). However, the current approach, even when it is successful, leads to an incomplete understanding of measurement. The study of measurement theory (in the sense offered in this paper) and the search for systematic errors that can be guided by that theory may contribute to broaden the understanding of physical measurement and to inspire a more adequate education of future scientists.

Resources for studying measurement theory

This paper presented some general information concerning Helmholtz' theory of measurement and attempted to motivate further study and use of that approach. Any person interested in studying and applying this theory in physics education will need additional literature on the subject. What can I suggest?

Helmholtz' work was a landmark, but it is not the best presentation of his approach. It can be used, but one should keep in mind its several limitations, such as the ones that were described above.

Norman Campbell's presentation of measurement theory is remarkably clear, well organized, and written in an accessible style. It is more complete than Helmholtz' original essay and I do warmly recommend it. Although Campbell's *Foundations of science* was originally published almost one century ago, it is still very useful as an introduction to measurement theory, for physicists. It is available as a reprint (Campbell, 1957).

Brian Ellis, a philosopher, published in 1968 a very useful book on *Basic concepts of measurement* (Ellis, 1968). His work was not as rigorous as later developments in measurement theory, but in some sense that is an advantage: many recent books and papers on measurement theory are hermetic and can be described as scientifically sterile. They would not appeal to a scientist wanting to improve his scientific research or his teaching. Ellis' book is well written and, although the author was a philosopher, he did present the main ideas in such a way that physicists and other scientists will have no difficulty in understanding and enjoying most of the book.

I would not recommend to physicists and physics teacher the three volumes of *The foundations of measurement*, the current "Bible" of measurement theory (Krantz, Luce, Suppes & Tversky, 1971-1990). This fundamental treatise is a very nice contribution to the current view of measurement, but its style does not seem suitable for most working scientists. Much of current publications on measurement theory only discuss the mathematical side (the axioms that the physical operations should mirror) or philosophical issues, but do not address the central point discussed here – the possibility of *testing* a measurement procedure by imposing that it should obey some rules analogous to those of arithmetic operations.

There are many different issues related to measurement theory. Some of them (such as those discussed in the present paper) can be perceived as directly relevant to the scientific practice, having important consequences for the practice of experimental physics. Other issues, although they have a strong appeal to measurement theory experts, are more remotely associated to the practice of experimental science. From the philosophical point of view, there are relevant *ontological issues*: Do physical objects *have* quantitative properties, or are quantities something extrinsic to physical objects? Some papers present a nice account of several epistemological problems of measurement – most of them having little relation to the scientific experimental practice (Mari, 2003; Boniolo, 2002). Therefore, the direct relevance of a large part of that literature to the practice of the physics laboratory is not very high.

Concluding comments

This paper maintains that a specific part of philosophy of science can be used to improve the understanding and practice of measurement and that for that reason it would be valuable to introduce it in science education.

Sometimes the use of philosophy of science in education is viewed as a mere transmission of the views adopted by some specific philosophers, with no regard to the specific needs of science education. I agree that philosophy is useful and should be taught, but sometimes it may fail improving science education. Borrowing a phrase coined by Jorge Paruelo, what is urgently needed is an *applied philosophy of science* (Paruelo, 2003, p. 334), selecting and adapting the philosophical knowledge (and techniques) to science teaching. The teaching of the elements of measurement theory in the physics laboratory may be a fine step in this direction. Helmholtz' approach to measurement can be easily taught in undergraduate physics courses. We introduced it successfully in Brazil, several years ago. This theory of measurement provides a more adequate basis for the physical laboratory than operationalism and may improve the students' understanding of experimental physics.

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