Absolute synchronization by light beacons: A paradox

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The light pencil emitted by a rotating beacon may produce at great distances a spot of light moving with superluminal speed. This light spot may be used for synchronization of distant clocks, and this method would seem to provide an absolute criterion for simultaneity due to an invariant feature of rotation. This conjecture is explored and then contrasted with another argument that denies that such synchronization is absolute. The shortcomings of the first argument are discussed, and it is shown that this process is equivalent to the usual synchronization by light signals.

I. INTRODUCTION

The main reason for preferring Einstein's special relativity to Lorentz's ether theory was epistemological.[1] The all-important testable predictions were the same for both theories. According to Lorentz's theory, the speed of light is constant relative to one single preferred referential frame: the ether frame. According to Einstein's theory, its speed is constant relative to any inertial referential system. Although the difference is very important and clear, no experiment has been able to discriminate between these ideas. In order to test the isotropy of light propagation it is necessary to devise an "absolute" method of clock synchronization that must be independent of the assumption of isotropy of light propagation. Such a method has not been hitherto found.[2]

In this paper a new synchronization method is shown. At a first glance, it seems that it would produce an absolute synchronization of distant clocks. A second analysis shows that the method will always produce results equivalent to

Einstein synchronization. The paradox is analyzed, and the error in the first description of the method is found. It is concluded that this method is not independent of the usual one, and that it does not allow a crucial test between relativistic and ether theories.

II. FIRST ARGUMENT: SYNCHRONIZATION BY A LIGHT BEACON

If an opaque body moves near to a light source, its shadow projected on a distant wall may move with superluminal speed. The same will hold for the light spot projected by a rotating light beacon.[3] There are no difficulties in the production of such superluminal light spots. The beam from a powerful laser is visible at great distances; this light may be reflected near the source by a rotating mirror with a rotation frequency greater than 10^4 Hz. At a wall placed at a distance of 100 km, the light spot will be moving at a speed greater than 6×10^6 km/s (twenty times greater than c). In general, the tangential speed u of the light spot at a distance R from the mirror, which rotates at an angular speed ω, will be

$$u = \omega R . \tag{1}$$

Could such a high speed be used to establish absolute synchronization? There is something attractive in this idea, because, unlike previous suggestions, there is something absolute (invariant) in rotation. While the direction of a translation is a completely relative feature, the *orientation* of a rotation does not depend on the motion of the rotating body relative to a referential system. If two referential systems have parallel and equally oriented axes, then the orientation of the rotation of a body relative to these systems will be the same, although the period of rotation will in general be different. Put briefly: velocity transformations may change the sign of the velocity of a body; but transformations of periods and angular motion do not change signs. In this sense, rotation has something absolute, and hence we may expect that the motion of the associated light spot will also have invariant properties that may allow its use for absolute synchronization. Let us remember that rotating bodies sometimes introduce problems that cannot be adequately solved by special relativity alone, and hence we are allowed to think that the rotating beacon might be something outside the scope of conventional relativity. Let us see how could it be used for synchronization between two distant clocks.

Consider two distant clocks A and B on the surface of a nonrotating planet, as shown in Fig. 1. A very powerful laser beacon is built and sent away in a rocket, in a direction perpendicular to the medium point of the straight line AB. The axis of rotation of the light pencil is kept invariant in a definite direction (e.g., pointing towards some star) and is perpendicular to both AB and the direction of rocket motion. The light beam reaches B and then A in each turn. The orientation of the rotation of the beacon is known, and hence it is also known which of the two points A and B is reached before. Relative to any other referential system, this rotation will have a different period but the same orientation, and therefore all observers will agree that the light beam passes through B before it reaches A. The distance between the rocket and the planet can also have no influence on the orientation of rotation. Hence, at any time after the launching of the rocket we know how the light pencil is turning.

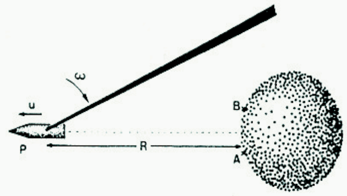

Fig. 1. Light beacon mounted upon a rocket P recedes from a planet with speed u. The light pencil emitted by the beacon turns with angular speed ω, and scans the surface of the planet from B to A with tangential speed equal to ωR. As the distance R of the rocket increases, this speed will increase beyond any limit. The time lag between the passages of the light pencil through A and B will decrease and become negligible, hence providing an asymptotic synchronization criterion.

As the distance of the rocket increases, the speed of the light spot moving from B to A at the surface of the planet will also increase, according to Eq. (1). Hence, we have a phenomenon with invariant direction and increasing speed. These properties may be described in a mathematical formalism. Let us denote $t_{(PA)n}$ the instant when the light from the rocket-borne beacon P reaches A for the nth time (at the nth turn). The symbol $t_{(PB)n}$ has an analogous meaning. The above argument has shown that

$$t_{(PB)n} < t_{(PA)n} , \tag{2}$$

$$\lim_{n\to\infty} (t_{(PA)n} - t_{(PB)n}) = 0 . \tag{3}$$

The use of this light beacon provides us with an asymptotic criterion for simultaneity: as the distance of the rocket increases, the time interval between the passages of the light pencil by B and A becomes smaller than any positive quantity, and therefore the difference between these times will become negligible compared with any chosen limit of error, if we wait till the distance of the rocket is sufficiently large. With the use of two rockets furnished with similar beacons the method may be improved, as will be shown below.

III. CHECKING THE METHOD

The use of light beacons provides an "operational definition" of simultaneity at two distant points. As a kind of definition, it could seem that it is completely arbitrary and that one may accept or refuse it at his will. This is not correct. The method was grounded on some assumptions that have a lot of consequences. Some of them can be tested, and those tests may either show that the underlying ideas are wrong or that they are acceptable.

Let us suppose that another beacon Q similar to P is built and sent simultaneously to space in another direction, obeying the same perpendicularity conditions described for P: the rocket moves in a direction perpendicular to the medium point of the straight line AB; the axis of rotation of the light pencil is perpendicular to both AB and the direction of rocket motion. But let us suppose that Q turns in the opposite orientation, in such a way that its light is made to reach A before B. Relations analogous to Eqs. (2) and (3) will hold:

$$t_{(QB)n} > t_{(QA)n} , \tag{4}$$

$$\lim_{n\to\infty} (t_{(QB)n} - t_{(QA)n}) = 0 . \tag{5}$$

In order to simplify the argument suppose that the light pencils from P and Q always strike A at the same time:

$$t_{(PA)n} = t_{(QA)n} . \tag{6}$$

From relations (2) to (6) we derive

$$t_{(QB)n} > t_{(PB)n} , \tag{7}$$

$$\lim_{n\to\infty} (t_{(QB)n} - t_{(PB)n}) = 0 . \tag{8}$$

Equations (7) and (8) describe observable consequences of the general assumptions of this synchronization method. Both portray relations between events that occur at the same place B and can therefore be tested without any assumption about distant simultaneity. Relation (7) states that the light from P will always reach B before the light from Q, at each turn of the beacons. If the beacons have different colors, this may be easily tested. Even if this is not the case, remember that the rockets are sent in different directions, and may easily be distinguished.

Relation (8) states that the time interval between these events tends to zero. Both these predictions may be experimentally tested, and the experiment may refute or confirm the assumptions of the method. Besides, we may also deduce

$$t_{(QB)n} - t_{(QA)n} < t_{(QB)n} - t_{(PB)n} , \tag{9}$$

$$t_{(PB)n} - t_{(PA)n} < t_{(QB)n} - t_{(PB)n} . \tag{10}$$

These equations state that if at any instant we assume the approximation

$$t_{(QB)n} = t_{(QA)n} \text{ or } t_{(PB)n} = t_{(PA)n}$$

for synchronization purpose, the maximum error will be the observable difference $t_{(QB)n} - t_{(PB)n}$. It is also obvious that the best approximation will be given by

$$t_{(QA)n} = \tfrac{1}{2}(t_{(QB)n} + t_{(PB)n}) . \tag{11}$$

This method of synchronization is independent of Einstein's method, and makes use of superluminal phenomena. It may be used in the measurement of one-way speeds of light therefore providing a crucial test for choosing between Einstein's and Lorentz's theories. If the former is true, then the above method must always yield results equivalent to those of light synchronization. If Lorentz's theory is true, then the coincidence of the results of the two methods will occur only if the clocks are at rest relative to the ether. In the latter case the comparison between the two methods would allow the measurement of absolute speed (speed relative to the ether) of referential systems.

IV. SECOND ARGUMENT: CRITICISM OF THE METHOD

Another argument shows that synchronization by light beacons is not absolute: some observers would not agree that clocks synchronized by this process are really synchronous.

Consider a referential system S', and suppose that relative to this frame the planet moves with a velocity v parallel to the straight line AB as shown in Fig. 2(a). At each instant the rocket lies in the straight line perpendicular to the middle point of AB. But light does not propagate instantaneously from the rocket to the planet: it takes some time T to travel this distance, and in this time interval the planet moves to a new position. Consequently the light from the rocket will not reach the planet from a direction perpendicular to AB: it will arrive at an angle θ with that direction. In the time T the planet has moved a distance $MM' = vT$. In this same time, light travels from the rocket to the planet, hence $OM' = cT$. From geometrical considerations it is evident that

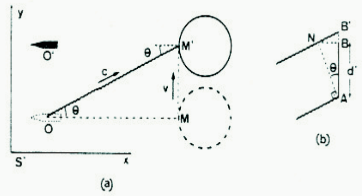

Fig. 2. (a) Relative to a referential system S' the planet has a velocity v parallel to AB. In this case, the light from the rocket will reach the planet from an oblique direction. (b) In this case the light from the rocket must reach A before B, and the time lag between these events will not vanish as the distance of the rocket increases.

$$\sin\theta = MM'/OM' = v/c . \tag{12}$$

If the motion of the planet is directed from A towards B, the light from the rocket will reach A before reaching B, since light reaches the planet from a point that is nearer to A than to B. Relative to S', the difference between $t'_{(PA)n}$ and $t'_{(PB)n}$ will not tend to zero, but will approach a finite value, as will be proved below.

When the rocket is very far from the planet, its light reaches both A and B from nearly parallel directions as sketched in Fig. 2(b). When the light reaches A, it is still at a point N distant from B. Between this instant $t'_{(PA)n}$ and the instant when light reaches B the planet is still moving. Therefore, in this time interval $\Delta t'$ the point B will reach another position B'. The extra distance traveled by light is $NB' = c\Delta t'$. From geometrical considerations it is easily seen that

$$NB' = NA \tan\theta , \tag{13}$$

$$NA \cos\theta = AB . \tag{14}$$

Hence we deduce

$$\Delta t' = NB'/c = (AB/c)(\sin\theta/\cos^2\theta) ,$$

and therefore, using Eq. (12),

$$\Delta t' = (ABv/c^2)/(1 - v^2/c^2) . \tag{15}$$

Relative to S', there is consequently a finite time lag between the instants when the light from the rocket reaches A and B. When the distance of the rocket from the planet is very great, this time lag approaches the value shown in Eq. (15). Hence, according with this second argument, not every observer will agree that the rotating light beacon provides an adequate synchronization of the clocks at A and B. Hence it seems that this method does not provide a criterion for absolute simultaneity. Observe that, if we substitute the contracted distance $d' = d_0(1 - v^2/c^2)^{1/2}$ for AB, we obtain the usual equation of time lag between clocks synchronized by light flashes:

$$\Delta t' = (d_0 v/c^2)/(1 - v^2/c^2)^{1/2} . \tag{16}$$

According to this second argument, synchronization by light beacons is equivalent to synchronization by light flashes (Einstein synchronization).

V. COMPARING THE ARGUMENTS

The contrast between the conclusions of Secs. II and IV presents a problem. The two arguments cannot be both correct. Both seem plausible when studied independently; but the paradox may be solved by showing that one of the arguments is wrong. When this error is eliminated both analyses lead to the same conclusion.

The first argument (Sec. II) assumed two basic ideas: (i) the rotation of the light beacon is qualitatively invariant; (ii) the distance between the rocket and the planet increases, and this produces an increasing and limitless speed of the light spot from B to A. In the second argument the rotation of the light pencil is not mentioned. It is implicitly assumed that light is emitted simultaneously towards both points A and B. This is not a wrong assumption, because the second argument analyzes the limiting case when R tends to infinite values. In this case the time difference between the emission of light towards A and towards B is negligible; the situation becomes equivalent to the use of a distant flashlight. Up to this point the two arguments seem to have compatible premises. But the second argument shows that as R tends to infinite values the difference between $t_{(PB)n}$ and $t_{(PA)n}$ does not approach zero, and that therefore the sweep speed does not tend to infinite when R increases. Either this idea or the second assumption of the first argument must be wrong.

If the speed of light is isotropic relative to the rocket, then Eq. (1) must be correct, relative to this system. From this relation, hypothesis (ii) of the first argument follows immediately, *if the use of other referential systems does not introduce changes in the conclusions*. Here is a problem. If the speed of light is isotropic relative to a system, it may be anisotropic relative to another system. And, after all, if we are trying to develop a *test* of the isotropy of light propagation, we cannot *assume* this in our arguments. We can at most suppose that light has isotropic propagation relative to one referential system, because this hypothesis is compatible both with Einstein's and Lorentz's theories. Does this wrong supposition invalidate any of the arguments?

In the second argument it is supposed that light travels in straight lines relative to S', and that its speed is the same in both paths from the rocket to A and B. But as the situation considered in the second argument corresponds to parallel light rays reaching both A and B, the equality of the speed of light along the two paths will hold even if light has anisotropic propagation relative to S'. It is only necessary to suppose that space is *homogeneous* for light propagation, in the second argument.

In the first argument, however, the assumption of isotropy of light propagation seems an essential requirement. If this hypothesis does not hold relative to the rocket proper system, then the *mean* tangential speed of the light spot at a distance R will be given by Eq. (1), but its value at any particular place may be different from this mean value. Let us also remark that the speed of the light spot along the straight line AB is not equal to its tangential speed at that point, relative to S', because in this case light reaches AB obliquely.

Even after those comments it is not easy to see what is wrong in the first argument. The beacon emits a continuous light pencil, and rotates in a definite and invariant way. This pencil always sweeps the planet from B to A. Even if this pencil is not perpendicular to AB, it must always reach B before A, relative to any referential system. Besides, if the distance between the rocket and the planet increases, the sweep speed must also increase in any direction, whatever the existing anisotropy of light propagation. Instead of Eq. (1) we may have something such as

$$u = \omega R f(\beta), \tag{17}$$

where β is an angular parameter characterizing the considered direction. What is wrong?

It is difficult to refute the first argument because it is almost completely qualitative. It makes inferences from an *image* of a turning light pencil, such as that shown in Fig. 1. If this picture is correct, the argument should be valid. What is wrong is the image itself. Figure 1 would be a reliable image if its application was restricted to slow rotation and small distances. But as the distance of the rocket increases and the sweep speed becomes comparable with c the light pencil will not remain straight. It will become curved, as will be proved below, and this is the main reason of the failure of the first argument.

VI. CURVED "PENCIL" OF LIGHT

At each instant the rocket sends light towards a different direction, as shown in Fig. 3(a). The light emitted in each direction propagates in straight paths, if the approximations of geometrical optics may be applied. But as the light recedes from the source, the emission direction changes, and the succeeding parts of the light pencil are sent towards other directions. Hence the whole pencil cannot be straight. It would be straight only if the rotation speed were zero or if the speed of propagation of light were infinite.

It is not difficult to derive the equation that describes the form of the light pencil relative to the proper referential system of the rocket, if light propagates isotropically relative to this system. Let η be a parameter that specifies the instant when a given fraction of light was emitted. The beacon turns with a constant angular speed ω. With a suitable choice of initial conditions the direction of emission β will be related to the instant of emission by the relation

$$\beta = \omega\eta . \tag{18}$$

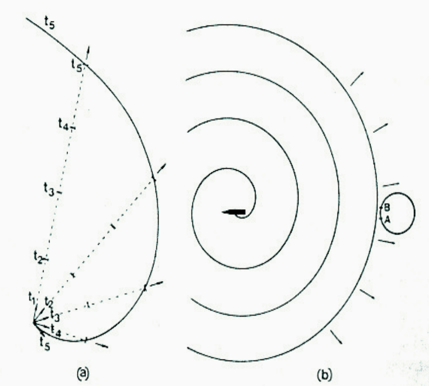

Fig. 3. Light "pencil" emitted by the light beacon is curved. (a) At each instant the beacon emits in a different direction. Each portion of light recedes from the beacon with constant velocity, forming an expanding curved light front. (b) When the rocket is at a very great distance, the light front approaching the planet is nearly circular, and reaches A and B at almost the same time. However, this simultaneity is not frame invariant.

At a further instant t this fraction of light will be at a distance r from the rocket:

$$r = c(t - \eta) . \quad (19)$$

At any instant the series of parts of the pencil will obey the relation

$$r = c(t - \beta/\omega) . \quad (20)$$

This is the equation of the so-called Archimedes's spiral. As the rocket recedes from the planet, the form of the light beam will become more and more curved [Fig. 3(b)]. As the distance R tends to infinity, the portion of the light pencil near to A and B will approach the form of a circular arc which will touch A and B almost simultaneously, relative to this system.

Relative to another referential system such that the rocket moves and light propagation is not isotropic this light spiral will be distorted. But even in this case the branch that arrives at the planet when the rocket is at a great distance will resemble a circular arc with center in the position where the rocket was when it emitted this light. If the planet has no velocity component parallel to AB, this curved branch will tend to become a straight light front parallel to AB, reaching both points simultaneously. If the planet has a finite velocity component parallel to AB relative to another referential system, the branch of the spiral that reaches the planet will tend to become a straight light front inclined relative to AB, and will reach these points with a finite time delay. This synchronization is similar to the synchronization by a rigid straight rod[4] and is as relative as that.

The wrong component of the first argument was therefore an image: the picture of a straight pencil of light turning around the rocket. This wrong "gestalt" must be replaced by that of a curved light front expanding radially from the rocket. The use of this second image is compatible with the second argument and shows that the first one was inadequate. This solves the paradox.

VII. CONCLUDING REMARKS

The second argument (Sec. IV) was written in a relativistic style, and doubts may remain about its conclusion if Lorentz's theory was adopted for the analysis of the process. But it may be easily shown that synchronization by light beacons is equivalent to Einstein synchronization, whatever the adopted theory. If the planet is at rest relative to the ether, both methods provide a "true" synchronization. If the planet moves relative to the ether, both provide "wrong" synchronizations. Read again the argument in Sec. IV, replacing the idea of the referential system S' by the idea of the absolute ether system. All the arguments will have the same form, and at last we shall infer that the method will not provide a "real" synchronization between A and B. Equation (16) in this case would give the systematic error of the method, and this is quantitatively equal to the systematic error of Einstein synchronization according with Lorentz's theory. Hence, if anyone would take this idea seriously, and use light beacons to synchronize distant clocks in order to test the isotropy of propagation of light, then both Einstein's and Lorentz's theories would predict that the measured speed of light would be isotropic, because this method is equivalent to Einstein synchronization, which assumes the isotropy of light propagation.

It would also be very instructive to consider the possibility of using other kinds of superluminal phenomena to synchronize distant clocks, and to compare these methods with Einstein's. No kind of real contradiction has ever been found in such attempts, and most of us hope that none will be found. However, it has not hitherto been proved that synchronization by superluminal phenomena must always lead to the same result as Einstein's process. Will someone find some day, in this or in some other field, a kind of phenomenon that would allow absolute synchronization and therefore a test of the isotropy hypothesis? It seems very important either to find it or to prove that it cannot be found.

ACKNOWLEDGMENT

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[1]There has been a recent controversy about the reasons that decided the acceptance of Einstein's and rejection of Lorentz's theory: E. Zahar, Brit. J. Philos. Sci. **24**, 95, 223 (1973); P. Feyerabend, Brit. J. Philos. Sci. **25**, 25 (1974); K. Schaffner, Brit. J. Philos. Sci. **25**, 45 (1974); E. Zahar, Brit. J. Philos. Sci. **28**, 195 (1977). There has also appeared the claim that Einstein's relativity was accepted mainly for sociological reasons: L. S. Feuer, Ann. Sci. **27**, 277, 313 (1971).

[2]Historical references about the problem of synchronization may be found in H. Arzelies, *La Cinématique Relativiste* (Gauthier-Villars, Paris, 1955), p. 64. An old suggestion of synchronization by slow transportation of clocks has been recently brought again to light, producing a wave of papers: B. Ellis and P. Bowman, Philos. Sci. **34**, 116 (1967); A. Grünbaum, Philos. Sci. **36**, 5 (1969); W. C. Salmon, Philos. Sci. **36**, 44 (1969); B. Ellis, Austral. J. Philos. **49**, 177 (1971); P. L. Quinn, Brit. J. Philos. Sci. **25**, 78 (1974); I. W. Roxburgh, Brit. J. Philos. Sci. **26**, 47 (1975); J. Leplin, Brit. J. Philos. Sci. **27**, 399 (1976); L. A. Beauregard, Philos. Sci. **43**, 469 (1976); B. Ellis, Philos. Sci. **45**, 309 (1978). Another recently proposed method for synchronization makes use of a falling rod that would strike two distant points at the same time: F. Jackson and R. Pargetter, Philos. Sci. **44**, 464 (1977); R. Torretti, Philos. Sci. **46**, 302 (1979); C. Giannoni, Philos. Sci. **46**, 306 (1979); F. Jackson and R. Pargetter, Philos. Sci. **46**, 310 (1979). Other references and a discussion of one-way measurements of the speed of light may be found in O. Costa de Beauregard, Bull. Astron. **15**, 159 (1950); M. Rudefer, Am. J. Phys. **43**, 279 (1975); H. Erlichson, Am. J. Phys. **43**, 279 (1975).

[3]These well-known phenomena have been described as acceptable and nonproblematic: M. A. Rothman, Sci. Am. **203** (7), 142 (1960); E. F. Taylor and J. A. Wheeler, *Spacetime Physics* (Freeman, San Francisco, 1966), p. 71; R. G. Newton, Science **167**, 1569 (1970). Although the phenomenon is not new, the paradox described in this paper seems to be original. Its essential idea occurred to me in a dream, where a respectable physicist whose name I must omit described this "absolute" method of synchronization as a serious challenge to relativity. His exposition was so persuasive that it took me several days to find out what was wrong with the idea. Would Freud explain this dream as a symptom of hidden desire to overthrow the theory of relativity?

[4]See references on Footnote 2 above.