Physics Before and After Einstein
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Chapter 2

Mechanics and Electromagnetism in the Late Nineteenth Century: The Dynamics of Maxwell's Ether

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1. Introduction

The most important effects predicted by the theory of special relativity had already been obtained before Einstein's work. This fact is well known by historians of science, but not widely known among scientists and laymen.

In popular accounts of the theory of relativity, Einstein is usually depicted as the only person responsible for that theory. The physicists who had witnessed the rise of the theory knew otherwise, and when one consults early books on relativity (such as those written by Max von Laue, Wolfgang Pauli and other authors) it is possible to find that they acknowledged the central contributions of several other physicists to the theory. However, after a few decades the Einstein myth had established itself. The situation changed in 1953, when Sir Edmund Whittaker published the second volume of his book *A History of the Theories of Ether and Electricity*, which contained a chapter titled "Relativity theory of Poincaré and Lorentz" [120]. In that work, Whittaker minimized Einstein's contribution to the special theory and ascribed all relevant steps to Lorentz, Poincaré, Larmor and other physicists. This book triggered the publication of several papers discussing the relative contributions of Einstein and the other physicists who contributed to the building of special relativity.[1] Nowadays, historians of physics acknowledge that many other researchers before Einstein made very important contributions to relativity, although they have not reached a consensus about the relative importance of their works.

This chapter will present the development of those ideas, in the late nineteenth century and during the first years of the twentieth century – that is, before Einstein's first publication on the subject, in 1905. It will be shown that Maxwell's electromagnetism, together with the ether concept, which played a central role in his theory, led to a new dynamics where mass increased with velocity, and energy changes were accompanied by mass changes.[2]

[1] See, for instance, [15,100,30–32,98].

[2] Hirosige [46] studied the role of electromagnetism and ether in the rise of relativity theory.

In order to condense the subject into one chapter of suitable size, it was necessary to select specific topics to describe. It has been impossible to include here a general overview of the history of electromagnetism.[3] Therefore, many relevant subjects – such as Weber's electrodynamics – have had to be left out. Only Maxwell's field approach and some of its consequences will be dealt with here, giving special emphasis to the developments that led to results closely related to relativistic dynamics.

Another simplification adopted here is the use of modern notation and terminology. The mathematical notation used in electromagnetism in the late nineteenth century was seldom the one that is used today [107]. Besides, the period under discussion was a time when several systems of units were used by different authors. A faithful rendering of the historical contributions should use the original words, notation and systems of units employed by each author. Unfortunately, that would increase the size and complexity of the present work beyond acceptable limits. The approach I have thus adopted in this chapter is to use contemporary notation and the international (MKS) system of units throughout. I apologize for this anachronism.

2. Faraday and the Lines of Force

The starting point of this story will be the concept of ether developed by Faraday and Maxwell. Michael Faraday's (1791–1867) experimental work on electromagnetism led him to accept that electric and magnetic forces are not direct forces at a distance, but forces carried by a physical medium, possessing mechanical properties.[4] Faraday usually described the electric and magnetic effects using the concept of "lines of force". According to Faraday, electromagnetic forces are carried by lines of force stretched between electric charges.[5]

In the prevailing teaching of physics (especially at a basic level) we still make use of lines of force. However, the contemporary use is not equivalent to Faraday's concept, because we do not conceive of those lines as substantial physical realities. Michael Faraday accepted that the electromagnetic field has physical reality and quantitative properties, even in the absence of matter. The same idea was later used by Maxwell, in his electromagnetic theory.[6]

Faraday ascribed the effect of electrostatic induction, for instance, to "inductive lines of force", not to a direct action at a distance [23, § 1164]. This inductive effect seemed to propagate from point to point, through the medium. Metallic shields can impede the action of this effect, and accordingly Faraday thought that the lines of force reached to surface of the grounded metal barrier and could not pass through it. On the other hand, inductive effects can be noticed behind the borders of a metallic plate connected to the ground, hence the inductive lines of force seemed to bend around the metallic border [23, § 1221].

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[3] An account of the history of electromagnetism in the nineteenth century can be found in [20].

[4] The evolution of Faraday's ideas can be found in [122, Chap. 10].

[5] The interpretation of Faraday's ideas is not straightforward, and they changed over time (see [81]). Here, his concept of lines of force is presented taking into account his later ideas.

[6] Hesse [45] presents a historical and philosophical discussion of the tension between the field approach and the action-at-a-distance approach.

Why did Faraday introduce this conception? The main reason seems to be the possibility of intervening in the electric and magnetic interaction. Gravitational effects behave as direct actions at a distance, in a straight line, without suffering any change when matter is interposed. Electric and magnetic forces, on the other hand, seem to propagate along curved lines, and are influenced by interposed matter. Therefore, they behave as influences that gradually propagate in space – a space filled with some substance – from one point to the next. Since 1832 Faraday believed that actions took some time to pass from one point to another, and he thought that it would be possible to measure their corresponding speed [6, p. 107].

To account for the bending, Faraday supposed that the lines of force repel each other, and for that reason they separate from one another and bend towards regions where there are no other lines [23, §§ 1224–5, 1231]. Therefore, Faraday's inductive lines of force had two main properties: they produced induction and electrostatic attraction along the direction of the line; and they produced a mutual lateral repulsion [23, § 1297].

Faraday used a similar model for the lines of magnetic force – indeed, it seems that the study of magnetism was instrumental in shaping his belief in lines of force [34]. The effects of attraction between magnets or parallel electric currents were accounted for by the tendency of the lines to contract along their lengths [23, §§ 3266–7, 3280, 3294]. The repulsion of two similar magnetic poles, or two parallel opposite electric currents, was explained by the tendency of the magnetic lines of force to repel each other transversely [23, §§ 3266–8, 3295]. The effects observed in ferromagnetic, diamagnetic and paramagnetic bodies were also explained by those properties of magnetic lines of force, under the assumption that they tend to concentrate more or less in different substances [23, § 3298].

Faraday clearly stated that the lines of magnetic and electric forces are real [23, §§ 3263, 3269], and discussed their physical nature. He suggested that they could correspond to a vibration of the ether, or a state of stress of the ether, or some other static or dynamical state of the medium [23, § 3263]. It seems that in most cases he favoured the idea of tension. However, in 1845, when he discovered the magneto-optical effect [108], where a beam of polarized light is rotated when travelling along the direction of a magnetic field, he convinced himself that the magnetic lines of force have a dynamic nature – those lines turn around their length [23, §§ 2162–75; 122, pp. 386–391].

Using these ideas, Faraday emphasized the importance of the intermediate field, to the detriment of the idea of action at a distance. The electric charges, currents and magnets produce lines of force, but the lines of force are responsible for all distant interaction.

3. Maxwell and the Energy of the Electromagnetic Field

James Clerk Maxwell (1831–1879) adopted Faraday's views, adding a mathematical treatment to the qualitative model. In his *Treatise on Electricity and Magnetism,* Maxwell took up the idea of lines of force, and stated that he would adopt a theory where electric action is a phenomenon of tension of the medium, or tension along the lines of force [76, §§ 47–8].

Because of this approach, Maxwell characterized the electromagnetic phenomena by physical magnitudes distributed in space, instead of forces concentrated into points. The idea of an electric and magnetic field distributed in space, even in the absence of matter, used up to the current day, is a vestige of Maxwell's ether.

According to Faraday's views, the intermediary lines of forces produced forces, but the mechanical properties were always ascribed to matter, not to ether. In Maxwell's theory, on the other hand, the medium – the ether – is endowed with momentum, potential energy and kinetic energy (see [103,104]). Before Maxwell, the electrostatic energy of a system of charges, in a space devoid of matter, was calculated taking into account the value of the electric potential *at places where each charge was*:

$$W_e = \frac{1}{2}\sum(e_i V_i). \tag{1}$$

Starting from this equation (§ 84), and following a mathematical analysis that had already been developed by William Thomson (1824–1907) [118], Maxwell proved that electrostatic energy can also be computed from the knowledge of the electric field in the whole space, without any explicit mention of the charges that generate the field or react to the field. Indeed, the electrostatic energy can be computed from:

$$W_e = \frac{1}{2}\int \varepsilon E^2 \mathrm{d}V \tag{2}$$

where the integral is computed over all the space surrounding the charges [76, § 99a]. The integrated quantity ρ_e can be interpreted as the density of the electric field energy:

$$\rho_e = \frac{\mathrm{d}W_e}{\mathrm{d}V} = \frac{\varepsilon E^2}{2}. \tag{3}$$

According to Eq. (2), all regions of space where there is an electric field contribute to the total energy of the system, even when the field is generated by a set of charges contained in a small region. This energy scattered in space is, according to Maxwell, essentially a form of elastic potential energy [76, §§ 630, 638], connected to the ether tension [76, § 110]. This is a mathematical rendering of Faraday's lines of force [76, § 109].

In the case of a spherical body with a superficial charge q and radius a, in the vacuum, the total electrostatic energy around the charge can be computed by integration over all the space:

$$W_e = \frac{q^2}{8\varepsilon_0 \pi a}. \tag{4}$$

The same result was obtained by the old approach, of course. Maxwell presented a detailed quantitative analysis of those tensions in order to compute electrostatic actions taking into account only *local* actions, that is, eliminating actions at a distance. The mathematical development of the theory led him to associate nine components of electrostatic tension to each point of the electromagnetic medium, reduced to six by imposing the absence of rotational effects. In the absence of matter they reduce to only three stresses [76, § 106]: a longitudinal attractive tension (along the electrical field) and a transverse pressure (perpendicular to the direction of the electrical field). The import of those stresses is the same, both in the longitudinal and the transverse directions, when the sign is omitted. Their value p is equal to the density of electrostatic energy. In the presence of a material medium, Maxwell obtained [76, § 631] an equation equivalent to this:

$$\rho_e = \frac{\mathrm{d}W_e}{\mathrm{d}V} = -\frac{1}{2}\vec{\mathrm{E}} \cdot \vec{\mathrm{D}} = \frac{\varepsilon}{2}E^2. \tag{5}$$

Maxwell presented an analogous treatment for the magnetic field. However, in this case he took as the fundamental sources of the field the electric currents (not the magnetic poles), and regarding the currents as the motion of electric charges, he described the magnetic energy as "electrokinetic energy" [76, § 634]. He also computed the magnetic energy corresponding to a set of electric currents and proved that this energy can be computed by integration of a function of the magnetic field in all space. The density of magnetic energy – which, according to Maxwell, is essentially kinetic energy [76, §§ 630, 636, 638] – is given by:

$$\rho_m = \frac{\mathrm{d}W_m}{\mathrm{d}V} = -\frac{1}{2}\vec{\mathrm{B}} \cdot \vec{\mathrm{H}} = \frac{\mu}{2}H^2. \tag{6}$$

Maxwell states that this magnetic energy "exists in the form of some kind of motion of the matter in every portion of space" [76, § 636]. Maxwell accepted Faraday's magneto-optical effect as evidence that there is a rotational motion of the ether around the lines of magnetic force [76, §§ 821, 831].

Lagrange's method in mechanics was widely used in the nineteenth century. Maxwell described this method [76, §§ 553–567] and then showed that it could be applied to electromagnetism [76, §§ 568–584]. He did not explicitly present the Lagrangean function describing the electromagnetic field. However, since he accepted that the electromagnetic field has both potential energy W and kinetic energy T, it is possible to write down a Lagrangean function $L = T - W$ describing this field.

The possibility of describing electromagnetism according to the Lagrangean formalism was interpreted by Maxwell and his contemporaries as strong evidence that it was possible to provide a mechanical theory for electromagnetism. Several authors (including Maxwell himself) had proposed detailed mechanical models of the ether. However, all such models had limitations, and for that reason Maxwell, in his *Treatise on Electricity and Magnetism*, chose to avoid any specific model. Instead, he adopted a general approach, stating that all electromagnetic phenomena could be understood as effects transmitted through the underlying ether, and calculating the general mechanical properties ascribed to that medium, without proposing detailed mechanical models for the ether [89, pp. iv–viii].

4. The Ether Stresses according to Maxwell

The magnetic interactions, according to Maxwell, are also associated with ether stresses [76, §§ 641–3]. The description of those stresses is highly complex, in the general case (in the presence of matter), but in a vacuum they reduce to a longitudinal attraction and a transverse pressure, both with the same absolute value p_m, equal to the density of magnetic energy ρ_m:

$$\rho_m = p_m = \frac{\mathrm{d}W_m}{\mathrm{d}V} = \frac{\mu H^2}{2}. \tag{7}$$

At several places, Maxwell emphasized that he was following Faraday's ideas. It was also following Faraday's suggestion that Maxwell developed the electromagnetic theory of light [76, § 782]. At this point, Maxwell made use of the theory of ether stresses to compute the pressure produced by an electromagnetic wave [76, § 792]. Suppose that a polarized plane electromagnetic wave propagates in the z direction. The electric and magnetic fields are perpendicular to one another and transverse to the propagation of the wave. Let us suppose that the electric field is in the x direction and the magnetic field in the y direction. If the total energy density of the wave is ρ, on average half of this energy will be electric and the other half magnetic. Therefore, the mean electric stresses will be:

$$T_{ex} = -\frac{\rho}{2}, \tag{8}$$

$$T_{ey} = T_{ez} = +\frac{\rho}{2}. \tag{9}$$

The mean magnetic stresses will be:

$$T_{my} = -\frac{\rho}{2}, \tag{10}$$

$$T_{mx} = T_{mz} = +\frac{\rho}{2}. \tag{11}$$

Adding those stresses, Maxwell obtained the resultant effect:

$$T_x = T_y = 0, \tag{12}$$

$$T_z = \rho. \tag{13}$$

Therefore, there is no resultant pressure in the x and y directions (perpendicular to the direction of wave propagation), and there is a pressure in the direction of wave propagation (z) equal to the total energy density of the wave.

Maxwell's *Treatise on Electricity and Magnetism* was published in 1873, containing the theoretical prediction that light should produce pressure. Independently, in 1876, Adolfo Bartoli (1851–1896) deduced a formula for the pressure of radiation, using a purely thermodynamic argument [14]. He showed that, if radiation did not produce a pressure, it would be possible to violate the second law of thermodynamics. Bartoli's argument was generalized by Ludwig Boltzmann (1844–1906) and Prince B. Galitzine (*c.* 1840–1916), some years later [10,28]. The coincidence between the equations obtained by the thermodynamic argument and Maxwell's theory of ether stresses led to strong confidence in those results.

The later confirmation of this effect in 1901 by Pyotr Lebedew (1866–1912), Ernest Fox Nichols (1869–1924) and Gordon Ferrie Hull (1870–1956), was no surprise [59,83,84].[7] As a matter of fact, this result is not a specific property of electromagnetic waves. Any wave carrying energy (such as sound or water waves) will also produce pressure [94].

In Maxwell's mature theory, therefore, the ether had several mechanical properties. It amassed energy, it produced forces, it could produce pressure. To ascribe motion and momentum to the ether was the next step.

[7] Worrall [124] describes the history of the search for the pressure of light.

5. The Momentum of the Magnetic Field

Following Faraday once more, Maxwell remarked that the phenomenon of self-induction exhibited properties similar to inertia [76, §§ 546–7]. When an electric current flows through a circuit, and the electromotive force which produces that current is interrupted, the current does not immediately stop; it has a tendency to continue for some time. The resistance of the current to stop depends on both the value of the current and the geometric configuration of the wires through which it is flowing, which can be described by a quantitative property – the self-inductance of the circuit. If this effect depended only on the value of the current and the length of the wire, it would be possible to ascribe it to the inertia of the particles that carry the electric charge (note that, at the time, the electrons were unknown). However, when the same wire, carrying the same electric current, is bent or transformed into a coil, its self-inductance changes [76, §§ 548–9]. Therefore, the self-inductance cannot be interpreted as a property of the electricity carriers. How then can it be interpreted?

Maxwell provided several interpretations, all of them involving the dynamics of ether. On one hand, it was possible to account for the effect taking into account the energy of the magnetic field around the wires [76, § 552] – the energy depends on the geometric configuration of the circuit. On the other hand, Maxwell associated the magnetic field with ether motion; and therefore, the self-inductance could be associated with some dynamic effect around the wires, which Maxwell called "electrokinetic momentum" [76, §§ 578, 585]. Maxwell proved that it was possible to evaluate this momentum by integrating the vector potential around the circuit [76, § 590]. The vector potential could be interpreted as a momentum associated to the ether.

The path that led Maxwell to this concept of the vector potential is very interesting (see [11]). Faraday had interpreted the electromagnetic induction as due to changes of "the electrotonic state" – an obscure concept he introduced in the 1830s. Inspired by a paper published in 1847 by William Thomson, in 1856 Maxwell was able to represent the induced electromotive force at each point of a circuit in very simple form [73], which can be represented in modern notation as:

$$\vec{E} = \frac{d\vec{A}}{dt}. \tag{14}$$

In this paper, Maxwell interpreted the vector $\vec{A}$ as the "electro-tonic function", or "electro-tonic intensity". In the same paper, he clearly established the relation between this function and the magnetic induction $\vec{B}$. In modern notation,

$$\vec{B} = \vec{\nabla} \times \vec{A}. \tag{15}$$

Several years later, Maxwell gave the same vector another name: "electromagnetic momentum" [75]. The new name was probably inspired by Eq. (14) because, when we multiply both sides by an electric charge, the left side of the equation becomes a force, and according to Newton's second law the right side should be the time derivative of the momentum. Afterwards, in his *Treatise*, Maxwell introduced a third name for the same quantity: "vector potential" – the name we use nowadays [76, § 405].

Note that here Maxwell is ascribing a new dynamical property to the ether: momentum. In the presence of a magnetic field – or, more exactly, in the presence of the vector potential – a charge will have a dynamical momentum, *even if it is at rest.*

Maxwell's electrodynamics is essentially macroscopical, in the sense of dealing with electric charge and currents as continuous entities. However, the atomic theory of matter, associated with the study of electrolysis by Faraday, led to the idea that electricity is also atomistic. This provided motivation for the study of a theory involving the motion of charged particles.

6. The Energy of a Moving Charge: J.J. Thomson

In 1881, J.J. (Joseph John) Thomson (1856–1940) studied the electric and magnetic fields associated with an electrically charged particle moving in the vacuum [112].[8] His approach was valid only in the case of velocities much smaller than that of light. He supposed that the electric field was not changed by the motion of the charge. However, the motion of the charge is equivalent to an electric current, and it therefore generates a magnetic field. In modern notation, the magnetic field $\vec{H}$ can be computed (for low speeds) using the formula:

$$\vec{H} = \vec{v} \times \vec{E}, \tag{16}$$

where $\vec{v}$ is the velocity of the particle and $\vec{E}$ is the electric field created by the moving charge.

If the electric field did not change with the motion of the charge, its energy was also independent of the speed. However, the magnetic field generated by the motion of the charge was associated with an energy density proportional to the square of the magnetic field, according to Maxwell's theory. As this magnetic field is proportional to the speed, the additional magnetic energy associated with the moving charge will be proportional to the square of the speed. A detailed computation (not shown here) provides the following result, for a spherical charge:

$$W_m = \frac{q^2 v^2}{12\varepsilon_0 \pi a c^2}, \tag{17}$$

where a is the radius of the particle, v is its speed, and q is its electrical charge (supposed to be distributed on its surface).

This magnetic energy could be regarded as kinetic energy in two different senses. First, because Maxwell and his followers regarded magnetic energy as kinetic energy of the ether. Second, because in this case, the magnetic energy associated with a moving charge was proportional to the square of the speed, exactly like the usual mechanical formula for the kinetic energy K of a particle:

$$K = \frac{mv^2}{2}. \tag{18}$$

Equation (17) can be rewritten to become similar to Eq. (18):

$$W_m = \frac{1}{2}\left(\frac{q^2}{6\varepsilon_0 \pi a c^2}\right) v^2. \tag{19}$$

[8] Thomson's paper contained a small mistake which was corrected some months later by George Francis FitzGerald (1851–1901). See [24].

Thus the expression between parentheses could be regarded as an "electromagnetic ıass":

$$m_e = \frac{q^2}{6\varepsilon_0 \pi a c^2}. \tag{20}$$

Comparison with Eq. (4) shows that the electromagnetic mass m_e is proportional to ıe electrostatic energy W_e of the charge. Indeed, they have the following relation:

$$m_e = \frac{4}{3}\frac{W_e}{c^2}. \tag{21}$$

This formula (not presented by Thomson, but easily obtained from his result) is a pecial case of the mass–energy relation $m = E/c^2$. The numerical factor 4/3 will be iscussed later.

Thomson supposed that the charged sphere had a mechanical mass M, as well as its lectromagnetic mass. Therefore, its total kinetic energy would be:

$$K = \frac{1}{2}\left(M + \frac{q^2}{6\varepsilon_0 \pi a c^2}\right)v^2. \tag{22}$$

Note that the electromagnetic mass is not inside the particle; it is outside it, spread ll over the ether, and occupying an infinite volume.

In 1895, Joseph Larmor suggested that matter could simply be a collection of elec- ical particles, and that, in that case, all inertia would be of electromagnetic origin.[9]

The ether is now full of mechanical properties: force, pressure, potential energy, inetic energy and electromagnetic mass. Electromagnetic mass is not the mass of the ther itself. It is the mass associated with a change in the ether – the magnetic field roduced by a moving mass.

As we have seen, this concept of electromagnetic mass was derived by comparison ith the formula of kinetic energy. This is a specific concept that could be called "kinetic ıass", or "capacity of electromagnetic kinetic energy", if we adopt the term later pro- osed by Henri Poincaré (1854–1912) and Paul Langevin (1872–1946).[10] We shall soon ee that there are other relevant mass concepts.

One should also bear in mind that the electromagnetic mass obtained by Thomson nd Fitzgerald is a first approximation, valid only for small speeds, and that they were ware of this limitation.

. Electromagnetic Mass of Fast Moving Charges

n 1889, Oliver Heaviside (1850–1925) obtained exact equations for the field around a ıoving charge, using an elegant operational approach [38]. However, this new mathe- ıatical technique was not regarded as reliable at that time. For that reason, J.J. Thomson educed again the same results [113,114], confirming Heaviside's results. The electro-

[9] On Larmor's contribution to the theory of relativity, see [55,18].

[10] It is possible to define inertial mass in several ways, and the various definitions lead to different equations, ı the case of the theory of relativity (and in the case of the electromagnetic theory described here). See [57].

8. Energy Flux and Momentum in Electromagnetic Fields

The concept of electromagnetic mass was born from the study of the magnetic energy associated with a moving charge. Another approach was the study of the electromagnetic *momentum* associated with a moving charge. It was shown above that Maxwell ascribed a momentum $q\vec{A}$ to a charge q moving at a place where the vector potential is $\vec{A}$. This, however, is a momentum independent of the velocity of the charge, and therefore is not due to its motion. There is another kind of electromagnetic momentum, associated with the motion of the charge, that will be explained below.

Let us first introduce the concept of the flux of electromagnetic energy. This concept was proposed independently by John Henry Poynting (1852–1914) in 1884, and Heaviside in 1885 [93,37,39]. The main result they obtained can be expressed in modern notation as:

$$\vec{S} = \vec{E} \times \vec{H}. \tag{25}$$

That is, the flux of electromagnetic energy $\vec{S}$ (today called "Poynting's vector") depends on both the electric and magnetic field, and is perpendicular to both. In the case of an electromagnetic wave, for instance, both electric and magnetic fields are perpendicular to the direction of propagation of the wave; the flux of energy, of course, has the direction of propagation of the wave.

If the electric and magnetic fields are parallel, there is no flux of energy. If they are not parallel, there is a flux, even if the fields do not vary. For instance, if a charged capacitor is put between the poles of a magnet, with the electric and magnetic fields in perpendicular directions, there will be an energy flux flowing inside the capacitor, perpendicular to both fields. This is an unexpected result.

It is not possible to *prove* that the flux of energy is given by Poynting's equation. There are other formulae compatible with the basic equations of electromagnetism, as proved by Ritz 20 years later [95]. However, most authors understood at that time (as they do today) that the Poynting vector is the simplest and most acceptable equation for the flux of electromagnetic energy.

The simplest interpretation of the energy flux, in the late nineteenth century, was that it was somehow related to the motion of the ether due to the electric and magnetic fields carrying the electromagnetic energy. This suggested that, if the ether has mechanical properties, it should also have a momentum whenever the Poynting vector is not null.

J.J. Thomson introduced the concept of an electromagnetic momentum associated with the electromagnetic field in 1893 [115, Chap. 1; 117]. According to him, wherever there is an energy flux $\vec{S}$ there is also a density of electromagnetic momentum $\vec{g}$ proportional to the Poynting vector:

$$\vec{g} = \frac{\vec{S}}{c^2}. \tag{26}$$

Today, this formula is interpreted as a description of the momentum density of the electromagnetic field in empty space. At that time, electromagnetic fields were regarded as special conditions of the ether, and accordingly Thomson's proposal should be understood as the description of a new physical property of the ether. The rationale for introducing a momentum associated with the ether was very simple and convincing. The ether can produce forces upon charged particles. Therefore, the particles acted on by the ether undergo momentum changes. If the ether did not have a momentum, this would violate the law of momentum conservation.

In 1894, Hermann von Helmholtz (1821–1894) suggested that the ether probably moves with a very large velocity in strong electromagnetic fields [41]. Three years later, W.S. Henderson and J. Henry attempted to observe the motion of the ether in electromagnetic fields, using an optical interferometer [42]. This experiment was different to and independent of the Michelson–Morley experiment, since it did not attempt to detect the motion of the Earth through the ether, but attempted to put the ether in motion, in the neighbourhood of the Earth, by applying a strong electromagnetic field and measuring an eventual change in the velocity of the light in that region. No effect was observed, although the experiment seemed able to detect a variation in the speed of light of about 10 m/s.

In theories where there is instantaneous direct action at a distance between two particles, this problem does not occur, because the total momentum of the system of particles is constant. However, in field theories – such as Maxwell's theory – the interaction between two particles is not instantaneous: each particle interacts directly only with the field – that is, with the ether – and the effect is propagated from one point to a distant one at the speed of light. Therefore, in field theories, the field itself must have a momentum, otherwise Newton's third law is violated.

9. Maxwell and the Motion of the Earth through the Ether

When Maxwell developed the theory of electromagnetic waves, he proved that the speed of those waves should have a speed equal to (or close to) the measured speed of light, and concluded that light was an electromagnetic wave.[11] When in 1887 Heinrich Hertz (1857–1894) produced short wavelength electromagnetic waves in the laboratory and showed that they had indeed the speed of light, Maxwell's theory was strongly confirmed [43]. The reduction of light and optics to electromagnetism required the unification of two ethers: the ether that had been hypothesized by the defenders of the wave theory of light (such as Fresnel), and the electromagnetic ether.

The two main theories of the light ether had been proposed by Augustin Fresnel (1788–1827) and George Gabriel Stokes (1819–1903), in the early nineteenth century. Fresnel put forward the theory of an ether at rest in all places of the universe [27]. Bodies would move through the ether (or, conversely, the ether was able to flow through all material bodies). Stokes suggested that the ether could move, as a viscous fluid, attaching itself to the surface of material bodies [109].

Fresnel's theory was highly successful. It explained the aberration of star light, explained the null result of experiments carried out by François Arago (1786–1853) and other researchers who tried to detect the motion of the Earth relative to this stationary ether, and correctly predicted the velocity of light in a moving liquid [82]. This was confirmed in 1851 by Hippolyte Fizeau (1819–1896), using interference phenomena to measure changes of speed of light in moving water [25].

[11] Before the development of his electromagnetic theory, Maxwell had already been led to believe that light was an electromagnetic wave. One of the arguments that led to this conclusion was dimensional analysis. See [17].

In order to account for the polarization of light, the luminous ether should be able to transmit *transverse* waves, such as those produced in an elastic solid (which are different from the *longitudinal* waves transmitted by gases, such as sound in the air). It was difficult to envisage how those properties could be combined with those of the electromagnetic ether, because magnetic energy was regarded as kinetic energy of the ether.[12] Maxwell did not address this problem. However, he accepted the idea of a stationary ether and wondered about the possibility of detecting the motion of the Earth relative to this ether [54].

At the time when Maxwell devoted his attention to this problem, there were several known optical experiments that had been unable to detect the motion of the Earth though the ether. Fresnel had proved theoretically that this motion cannot be detected by measuring the deflection of light by a prism, but Maxwell attempted this kind of experiment – and got no positive results. Stokes proved that light experiments using reflection and refraction could not detect any influence of the motion of the Earth through the ether, in Fresnel's theory, to the first order of v/c [110]. Assuming that the speed of the solar system through the ether was similar to the orbital speed of the Earth (about 30 km/s), one had $v/c = 0.0001$ (or 10^{-4}); therefore, no effect of this order of magnitude could be expected to occur in optical experiments.

However, there had been two experiments that had reported positive results in the search for the influence of the motion of the Earth upon optical experiments.

One of them was carried out by Fizeau in 1859, and amounted to a small rotation of the plane of polarization of light when it traversed a pile of glass plates. The rotation was different depending on if the light was travelling in the direction of the motion of the Earth or in the opposite direction [26]. The other experiment had been carried out by Anders Jonas Ångström (1814–1874) in 1865. He measured the deflection of light by a diffraction grating, and reported that this deflection was different when light travelled in the same direction as the Earth and in the opposite direction. In both cases, the effects were very small, and the experiment was difficult to reproduce. Eleuthère Élie Nicolas Mascart (1837–1908) repeated both experiments, and reported no positive effect [72].

Maxwell was aware of Fizeau's and Ångström's experiments (however, he was not aware of Mascart's experiments), but thought that they were not decisive. He suggested two new experiments [54]: one based on the study of the eclipses of Jupiter's satellites; the other one was a proposal for experiments measuring the time taken by light to travel to and fro between the source, a mirror, and back to the source. In the first case, he expected a first-order effect, that is, about 10^{-4}. In the second case, he expected a second-order effect, that is, about 10^{-8} – a very small effect indeed. He thought that it was impossible, with current techniques, to measure such a small effect.

Maxwell consulted an astronomer, David Peck Todd (1855–1939), who told him that the available data on Jupiter's satellites were not precise enough to test the predicted effect. However, Maxwell's ideas stimulated a young experimental physicist, Albert Abra-

[12] Some contemporary authors emphasize that the concept of the ether, in the late nineteenth century, was contradictory and absurd. There were, indeed, serious difficulties, but there was no proof that the concept of the ether was contradictory, or that it was impossible to surmount those difficulties. On the other hand, those critics of the ether should be asked whether physicists confronted with contradictions should always reject their theories, and should instead devote some time to considering wave-particle duality, and wondering whether the usual interpretation of quantum mechanics does not pose more serious difficulties than the old ether theory.

ham Michelson (1852–1931), who decided to follow the second path suggested by him. This led to the famous interferometer experiment, a few years later.

The story of the Michelson–Morley experiment is well known [105]. Michelson attempted to measure the second-order effect of the motion of the Earth relative to the ether, assuming Fresnel's theory. His first optical interferometer was built in Berlin, and the measurements were made in April 1881. He was unable to observe the predicted regular shift of the interference fringes, and concluded that the hypothesis of the stationary ether (Fresnel's theory) was wrong. He insinuated that Stokes' theory could be the correct one. However, Michelson's first experiment was not reliable. The instrument was not sufficiently stable, and its sensitivity was too low. Besides that, Michelson's mathematical analysis was wrong, and was soon criticized by Lorentz (1886), Potier and other physicists.

Fresnel's theory had been strikingly confirmed by Fizeau's 1851 measurements of the speed of light in moving water. In 1886 Michelson, with the help of Edward W. Morley (1838–1923), repeated and confirmed Fizeau's experiment. Urged by Lord Rayleigh, Michelson and Morley decided to repeat the 1881 interferometer experiment. In 1887, with improved apparatus, they obtained a significant null outcome (irregular displacements, much smaller than the predicted theoretical displacement).[13] The result disagreed with Fresnel's theory, but neither Michelson and Morley nor most of the physicists of that time concluded that the ether did not exist. The experiment simply seemed a new constraint to be taken into account in constructing ether theories.

10. Lorentz, Poincaré and the Impossibility of Detecting Motion Relative to the Ether

Hendrik Antoon Lorentz (1853–1928) was the main physicist who attempted to reduce optics to electromagnetism, following Maxwell's theory. He adopted Fresnel's stationary ether, and studied the properties of light using electromagnetism together with an atomic model of matter [77,78]. To account for reflection and refraction, he assumed that matter contained charged particles ("ions") that could respond to electromagnetic waves.

Fresnel's theory had accounted for the null results of optical attempts to measure the speed of the Earth through the ether (to the first order of v/c). Lorentz decided to tailor Maxwell's theory in such a way that it would also predict the same electromagnetic phenomena (to the first order of v/c), independently of the motion of the system through the ether. Other authors, such as Joseph Larmor, in England, and Henri Poincaré also followed a similar path [18].

In a striking series of papers, those authors gradually built the electrodynamics of moving bodies. The main results were the set of field transformation equations, allowing the calculation of electric and magnetic fields relative to different reference systems; and the space and time transformation equations that, together with the field transformation equations, preserve Maxwell's equations in reference systems moving relative to the ether.

Those equations were not found immediately. Lorentz first attempted to find only first-order equations – that is, the electromagnetic counterpart to Fresnel's optical the-

[13] Later experiments, by Morley and Dayton C. Miller (1866–1941), produced small *positive* results [111].

ory [98]. However, when Michelson and Morley were unable to detect a second-order optical effect, the situation changed. Under the pressure of this experimental outcome, several authors began to suspect that the negative results had a deeper meaning. Henri Poincaré suggested that all future attempts, of any kind, would also be unable to detect the motion of the Earth relative to the ether, and that this result should be taken into account in all future theories. This is the essence of the principle of relativity [100]. After struggling for several years, at last Lorentz developed the transformations of the electromagnetic field and the space and time transformations that preserved Maxwell's equation *exactly* in reference systems moving through the ether.[14] Those developments are described in Chapter 4.

11. Lorentz, Poincaré and the Momentum of the Ether

Let us now return to the problem of the dynamics of the ether.

In his first versions of the electrodynamics of moving bodies, up to 1895, Lorentz did not introduce the concept of electromagnetic momentum, because he accepted that the ether was always at rest [66]. Therefore, in his theory, the ether acted upon charged particles, but the particles did not act upon the ether, and therefore there was a violation of the conservation of momentum. In 1900, in a paper where he discussed this problem, Poincaré proved that, in order to maintain the principle of conservation of momentum, it was necessary to ascribe a density of momentum to the electromagnetic field, by following Eq. (26), that is, J.J. Thomson's formula [87].[15] According to Poincaré, it was necessary to accept that the ether (or something associated with the ether) is equivalent to a fluid endowed with mass, able to produce forces and to be put in motion, and capable of carrying momentum.

At first Lorentz did not accept Poincaré's analysis. However, the concept of an electromagnetic momentum was soon accepted by most theorists, and the ether acquired an additional dynamical property. In particular, Poincaré associated a momentum $p = E/c$ to a directed pulse of electromagnetic radiation. This momentum is another way of explaining the pressure produced by radiation – or, conversely, it can be deduced from the formula for radiation pressure. In the same paper, Poincaré also showed that the theorem of centre of mass would hold for a charged particle plus the ether if a mass $m = E/c^2$ was associated with the energy E of the electromagnetic field. He did not apply this relation to material bodies, however.[16]

Poincaré also introduced the concept of an angular momentum associated with electromagnetic fields where the Poynting has a non-vanishing rotational. The only relevant application seems to be the angular momentum of circularly polarized light – a property that was confirmed by experiment many years later [9].

[14] There were some mistakes in Lorentz's 1904 paper, such as his inadequate transformation of current density, which were corrected by Poincaré in 1905. Only Poincaré's version of Lorentz's theory was completely covariant.

[15] On Poincaré's contribution one might consult [15,30] and [29].

[16] Once Poincaré established this result, it would be very easy, employing the principle of relativity (which Poincaré had already proposed) to prove that the relation $m = E/c^2$ also applies to material bodies. See [47,61].

12. The Discovery of the Electron

The successful atomic theory of matter led Lorentz, in 1892, to propose an atomic theory of electricity. He suggested that neutral atoms (or molecules) were composed of one larger charged particle connected to a smaller particle of opposite charge. The smaller particle (called an "ion" by Lorentz) was responsible for several properties of matter, including refraction and emission of light.

In 1896 Pieter Zeeman (1865–1943) discovered the splitting of spectral lines of sodium when the source was placed in a strong magnetic field. Zeeman's effect was immediately explained by Lorentz using his theory. He was able to compute the order of magnitude of the ratio of charge to the mass of the hypothetical ions, and showed that they should have a negative charge.

At the same time, following an independent line of investigation, J.J. Thomson had measured the charge-to-mass ratio e/m of cathode rays [116] and obtained a result similar to Lorentz's value for the ion. Independently of Thomson, Walter Kaufmann also studied the deflection of cathode rays and obtained similar results [48]. The agreement between the two parallel developments led to the rapid acceptance of the existence of the "electron" (a name proposed by G. Johnstone Stoney) as an universal component of matter.

The theoretical studies previously described had shown that one should ascribe an electromagnetic mass to a moving charge. Instead of a generic charged particle, the next research attempted to improve the analysis of the electromagnetic mass of the electron.

In 1898 and 1900, Phillip Lenard (1862–1947) measured e/m for very fast electrons, with speeds up to $c/3$ [60]. Although his measurements were regarded as inconclusive, they seemed to reveal an increase of mass with speed. The "purely mechanical" mass of a particle was supposed to be independent of its speed, while the electromagnetic mass was a function of its speed.

In 1900, Wilhelm Wien (1864–1928) conjectured (as Oliver Lodge had already proposed) that perhaps all mass was of electromagnetic origin, and in that case the fundamental mechanical properties of matter would be reduced to electromagnetism [121]. Maxwell had shown that electromagnetism could be described according to Lagrange's approach, and this was understood as proof that it was possible to provide a mechanical foundation for electromagnetism. Conversely, around 1900, this was interpreted as proof that mechanics could be reduced to electromagnetism. Of course, this reduction was only possible if all mass was of electromagnetic origin.

It became imperative to improve Lenard's measurements, and to establish the theory of electromagnetic mass on solid grounds, in order to compare the theory to a experiment, to decide whether the electron had a "purely mechanical" mass, or whether its mass was solely electromagnetic.

13. Kaufmann's Measurement of the Mass of Fast Electrons

In 1901 Walter Kaufmann (1871–1947) published the result of measurements of e/m for very fast electrons (beta radiation emitted by a radium compound), with speeds between 0.8–0.9c [49]; see also [16]. He noticed a very clear increase of e/m (and, therefore, an increase of the mass of the electron) in the range of velocities he studied. The quantitative analysis presented by Kaufmann assumed that the electrons had a "real mass" m_0 (that is, a purely mechanical mass, independent of its charge) and an "apparent mass" μ (that is, its electromagnetic mass). He calculated the "apparent mass" from Searle's formula of the energy of a charged spherical particle:

$$W = \frac{q^2}{8\varepsilon_0 \pi R}\left(\frac{c}{v}\log\frac{c+v}{c-v} - 2\right). \tag{27}$$

However, the electromagnetic mass cannot be directly obtained from this formula by a simple comparison with the classical equation for the kinetic mass of a particle. Therefore, Kaufmann used a new approach. The kinetic energy of a particle is equal to the total work produced by the force that accelerates it. The change dW of the kinetic energy is therefore equal to the work $F\,dx$ produced by the force F when the particle undergoes a displacement dx. But $dx = v\,dt$, therefore $F = (1/v)(dW/dt)$.

Now, defining mass as the ratio between force and acceleration, $m = F/a$, and since $a = dv/dt$, we obtain:

$$m = \frac{F}{a} = \frac{dW/(v\,dt)}{dv/dt} = \frac{dW}{v\,dv}. \tag{28}$$

Applying this formula to Searle's equation, Kaufmann obtained the "apparent" electromagnetic mass:

$$\mu = \frac{q^2}{8\varepsilon_0 \pi R v^2}\left[-\frac{c}{v}\log\left(\frac{c+v}{c-v}\right) + \frac{2}{1 - v^2/c^2}\right]. \tag{29}$$

Comparison between the experimental data and this formula showed that they did not match. Kaufmann was therefore led to assume that only *part* of the electron mass was electromagnetic, and the other part was mechanical (or "real"). A good fit with the experimental data was obtained assuming that about one-third of the total mass was "apparent" (or electromagnetic) in the low speed limit.

14. The Momentum of an Electron

Kaufmann's theoretical analysis was unsound. His derivation of the mass of the electron from energy considerations could only be valid if he were studying the *longitudinal acceleration* of electrons – that is, their change of speed due to some force. All equations he used had a scalar form (that is, they did not take into account the *direction* of motion) and could only make sense if the speed changed, because of the relations used in the derivation shown above. However, in Kaufmann's experiments he only measured the deflection of electrons when submitted to forces perpendicular to their velocities. The comparison between theory and experiment required a different analysis, taking into account the momentum of the electrons.

In January 1902, a few months after the publication of Kaufmann's measurements, Max Abraham (1875–1922) published a new theoretical analysis of the dynamics of moving electrons [1].[17] He conceived the electron as a rigid sphere, with its charge ei-

[17] See also [2–4]. On Abraham's contributions, see [32,16].

ther spread on its surface, or homogenously distributed in its volume. Starting from the Thomson relation between density of momentum and energy flux (Poynting vector), and integrating for the whole space, Abraham was able to compute the momentum associated with a moving electron.[18] After lengthy calculations, he obtained, in the case of the superficial charge:

$$G = \frac{e^2}{8\varepsilon_0 \pi Rc}\left[\left(\frac{1+\beta}{2\beta^2}\right)\ln\left(\frac{1+\beta}{1-\beta} - 1\right)\right], \tag{30}$$

where $\beta = v/c$. This momentum is a vector, and its direction is the same as that of the velocity of the electron. If we represent by $\hat{v}$ an unit vector parallel to the velocity of the electron, then we have:

$$\vec{\mathrm{G}} = G\hat{v} = G\vec{\mathrm{v}}/c\beta. \tag{31}$$

Therefore, Eq. (30) can also be written as:

$$\vec{\mathrm{G}} = \frac{e^2}{8\varepsilon_0 \pi Rc^2}\left[\left(\frac{1+\beta}{2\beta^3}\right)\ln\left(\frac{1+\beta}{1-\beta} - 1\right)\right]\vec{\mathrm{v}}. \tag{32}$$

Abraham used Newton's second law in the form $\vec{\mathrm{F}} = \mathrm{d}\vec{\mathrm{p}}/\mathrm{d}t$. When the electron is submitted to an external force, its momentum can change in both magnitude and direction. When the force acting upon the electron is parallel to its initial velocity, only the magnitude of the momentum will change. In that case (longitudinal acceleration), we have:

$$\vec{\mathrm{F}}_{//} = \frac{\mathrm{d}\vec{\mathrm{G}}}{\mathrm{d}t} = \frac{\mathrm{d}G}{\mathrm{d}t}\hat{v} = \frac{\mathrm{d}G}{\mathrm{d}v}\frac{\mathrm{d}v}{\mathrm{d}t}\hat{v} = \frac{\mathrm{d}G}{\mathrm{d}v}\vec{\mathrm{a}}_{//}. \tag{33}$$

In this case (longitudinal force), $(\mathrm{d}v/\mathrm{d}t)\hat{v}$ is the longitudinal acceleration $\vec{\mathrm{a}}_{//}$, and $\mathrm{d}G/\mathrm{d}v$ can be interpreted as the electron mass. Abraham called it the "longitudinal mass".

If the force is perpendicular to the velocity of the electron (for example, for a magnetic force acting upon a moving charge), the speed and the value of the momentum will not change, but the motion will suffer a deflection and the path will be circular. In that case, we have:

$$\vec{\mathrm{F}}_{\perp} = \frac{\mathrm{d}\vec{\mathrm{G}}}{\mathrm{d}t} = G\frac{\mathrm{d}\hat{v}}{\mathrm{d}t} = \frac{G}{v}\frac{\mathrm{d}\vec{\mathrm{v}}}{\mathrm{d}t} = \frac{G}{v}\vec{\mathrm{a}}_{\perp}. \tag{34}$$

The transverse acceleration $\vec{\mathrm{a}}_{\perp}$ is the centripetal acceleration of the electron's circular motion, and G/v can be interpreted as the electron mass. Abraham called it the "transverse mass". From Eq. (32), we obtain for the longitudinal mass:

$$m_{//} = \frac{e^2}{8\varepsilon_0 \pi Rc^2}\frac{1}{\beta^2}\left[-\frac{1}{\beta}\ln\left(\frac{1+\beta}{1-\beta}\right) + \frac{2}{1-\beta^2}\right] \tag{35}$$

[18]In 1901, Lorentz had computed the momentum of an electron, but he used an approximation that was only valid for low speeds.

that is, Kaufmann's formula (29). And, for the transverse mass:

$$m_{\perp} = \frac{e^2}{8\varepsilon_0 \pi R c^2} \frac{1}{\beta^2} \left[\left(\frac{1+\beta}{2\beta} \right) \ln \left(\frac{1+\beta}{1-\beta} - 1 \right) \right]. \tag{36}$$

Both equations reduce to $e^2/(6\varepsilon_0 \pi R c^2)$ for speeds much smaller than c.

Using this analysis, Abraham showed that Kaufmann's experimental data should be compared to the transverse mass, because he measured the deflection of electrons. However, Kaufmann had used an equation equivalent to Abraham's longitudinal mass. Therefore, his analysis was wrong.

Kaufmann recognized his mistake, and in 1902 he published a new paper, with additional measurements analysed using Abraham's theory [50]. He concluded that there was good agreement between the experimental data and the transverse mass formula, and concluded that the entire mass of the electron was purely electromagnetic.

15. The Mass of a Box Full of Light

The experimental confirmation of the pressure of light in 1901 led to new theoretical work. In 1904, Max Abraham computed the pressure produced by radiation upon a *moving* surface, when the beam of light reaches the surface of a mirror in any angle [5]. Starting from Abraham's results, Friedrich Hasenöhrl (1874–1916) studied the dynamics of a box full of radiation [36].

Imagine a cubic box with perfectly reflecting internal surfaces, full of light. When the box is at rest, the radiation produces equal forces upon all those surfaces. Now, suppose that the box is accelerated, in such a way that one of its surfaces moves in the x direction. It is possible to prove that, when the radiation inside the box strikes this surface, the pressure will be smaller, and when it strikes the opposite surface, the pressure will be greater, than in the case when the box is at rest (or in uniform motion). Therefore, the radiation inside the box will produce a *resultant force* against the motion of the box. So, to accelerate a box full of light requires a greater force than to accelerate the same box without light. In other words, the radiation increases the inertia of the box. In the case when the radiation inside the box is isotropic, there is a very simple relation between its total energy E and its contribution m to the inertia of the box:[19]

$$m = \frac{4E}{3c^2}. \tag{37}$$

Note that here, as in the theory of the electron, there appears a numerical factor $4/3$. This is not a mistake. The relation between those equations and the famous $E = mc^2$ will be made clear later.[20]

Hasenöhrl also computed the change of the radiation energy as the box was accelerated. He proved that the total radiation energy would be a function of the speed of the box. Therefore, when the box is accelerated, part of the work done by the external forces is transformed into the extra radiation energy. Since the inertia of the radiation is

[19] Hasenöhrl arrived at a different result, in 1904, but his integration mistake was corrected by Max Abraham in the same year and acknowledged by Hasenöhrl in 1905.

[20] Fadner [21] describes several contributions concerning the establishment of $E = mc^2$.

In the case of Lorentz's formula, the development in series gives:

$$m_{\perp} = \frac{e^2}{6\varepsilon_0 \pi R c^2} \frac{1}{(1 - v^2/c^2)^{1/2}} \cong \frac{e^2}{6\varepsilon_0 \pi R c^2} \left(1 + \frac{1}{2}\beta^2 + \cdots\right). \quad (43)$$

Comparing Eqs (42) and (43), we see that, for low speeds (that is, when $v/c = \beta$ is much smaller than 1) Abraham's and Lorentz's formulas provide similar results.

The comparison between the theories and the experimental results was indirect. Lorentz analysed Kaufmann's experiment using his own theory, and concluded that the data was compatible both with his theory and with Abraham's.

17. Bucherer's Electron

Besides Abraham's and Lorentz's models of the electron, there were other possibilities, of course. In the same year when Lorentz published his results, Alfred Heinrich Bucherer (1863–1927) proposed another theory [12,13]. He assumed that the electron contracted due to its motion, as Lorentz had assumed, but he supposed that the *volume* of the electron remained constant. Therefore, the longitudinal radius of the contracted electron would become $L = R(1 - v^2/c^2)^{1/3}$ and its transverse radius becomes $L' = R(1 - v^2/c^2)^{-1/6}$, where R is the radius of the electron at rest.

Following the same basic theoretical ideas as Abraham and Lorentz, Bucherer obtained the following values for the transverse and longitudinal masses of the electron:

$$m_{//} = \frac{e^2}{6\varepsilon_0 \pi R c^2} \frac{1}{(1 - v^2/c^2)^{4/3}}, \quad (44)$$

$$m_{\perp} = \frac{e^2}{6\varepsilon_0 \pi R c^2} \frac{1}{(1 - v^2/c^2)^{1/3}}. \quad (45)$$

Independently of Bucherer, the same theory was proposed by Paul Langevin [56].

Other alternatives could also be envisaged. The charge of the electron could be spread over its surface or across its volume; the distribution of its charge could change, due to its motion, etc.

It seemed possible to test (by deflection experiments) which formula provided the best fit for the experimental data. It was also possible to provide theoretical arguments for choosing between the options.

In the next years, Kaufmann published new experimental data and compared his measurements to the three available theories of the electron [52,53]. He concluded that Abraham's formula provided the best fit. Max Planck (1858–1947), however, criticized Kaufmann's analysis [85], and concluded that his measurements were compatible with both Abraham's and Lorentz's equations – and that Lorentz's equation provided the best fit. At that time the situation was not clear [8,16]. Only ten years later, new experiments were able to confirm Lorentz's formula and rule out all other models.

18. The Contribution of Stresses to Mass

The experimental evidence was not sufficiently clear to provide a choice between the several models of the electron. Poincaré, however, provided a significant theoretical argument for Lorentz's theory.

In a paper written in 1905 and published the following year, Poincaré provided a review of Lorentz's theory and an analysis of the three theories of the electron [92]. He showed that Lorentz's theory should be supplemented by the assumption of a non-electromagnetic force holding the electron together. This force could be described as a negative pressure, of unknown origin. It was necessary to take into account this stress when computing the energy and momentum of the electron, and therefore the dynamics of the electron could not be derived solely from electromagnetism. However, after introducing this complement to Lorentz's theory, he proved that only Lorentz's theory was compatible with the principle of relativity. Conversely, if Abraham's or Bucherer's theory of the electron was valid, it should be possible to check, by measuring the dynamical properties of the electron, whether the Earth is at rest or in motion relative to the ether.

In Lorentz's theory, the electromagnetic mass of the low-speed electron should be equal to $m_0 = e^2/(6\varepsilon_0 \pi R c^2)$. The same result holds in the theories of Abraham and Bucherer. Now, the electrostatic energy of an electron at rest is $W_0 = e^2/(8\varepsilon_0 \pi R)$. Therefore, we can write the following relation between mass and energy of low-speed electrons:

$$m_0 = \frac{4W_0}{3c^2}. \tag{46}$$

For any contemporary reader, this relation seems very odd, because we are used to Einstein's equation $m = E/c^2$, without the numerical factor $4/3$. This difference was not due to any mistake made by Lorentz and the other theoreticians. It is an unavoidable consequence of electromagnetic theory.

However, there was one small blunder. Poincaré proved that electromagnetism could not provide a complete analysis of the electron. It was necessary to introduce other non-electromagnetic forces both to explain the stability of the electron and to produce a completely coherent dynamics (see [15]). Therefore, Poincaré was led to introduce a non-electromagnetic force that could be described as a negative pressure, counteracting the repulsive force produced by the charge at the surface of the electron. Taking this force into account, it is necessary to include a non-electromagnetic component in the equations of energy and momentum. This correction leads to a new relation between the total inertial mass m_0' and the total energy W_0'

$$m_0' = \frac{W_0'}{c^2} \tag{47}$$

which is compatible with the well known result of Einstein's theory.[22]

Most results of relativity theory were embodied in a paper Poincaré wrote in 1905, but this was published only the following year, in Italy [92].[23]

[22] Poincaré's approach is not accepted by all authors. F. Rohrlich and other physicists have criticized the introduction of Poincaré's stress and attempted to establish compatibility between electrodynamics and special relativity by a different route. It seems, however, that Poincaré was correct, and that we have two different viable approaches: Poincaré's and Rohrlich's. See [35].

[23] In 1905 Poincaré published a short note in the *Comptes Rendus* of the French Academy of Science, anticipating (without proof) some of the results of the larger paper. A lot of ink has already been used concerning the similarities and differences between Poincaré's contribution and Einstein's work. See Schwartz [99] (who translated a large part of Poincaré's 1906 paper); [79,62,29].

An argument similar to Poincaré's can be applied to the case of the box full of light. The mass associated to the light obeys a relation such as Eq. (46). However, this radiation produces pressure upon the walls of the box that contains it. The box is stretched by this pressure, and this tension must be taken into account when computing the mass of the whole system. For the complete system (box plus radiation), the relation between mass and energy obeys Eq. (47). This result was established by Max Planck, in 1907, after the publication of Einstein's theory ([86] see also [33]). In this fundamental paper, Planck proved that $m = E/c^2$ is not a general law. Indeed, it is valid for *closed* systems. However, any system submitted to an external pressure[24] will obey a different law: its mass will be proportional to its enthalpy $H = E + PV$, that is, $m = H/c^2$.

19. Conclusion

The development of Maxwell's electromagnetism, in his hands and in those of his followers, led to a new world view. On one hand, they provided a dynamical theory of the ether, showing that it was possible to ascribe forces, pressures, energy, momentum and mass to the electromagnetic field. The general dynamical relations were then applied to the electron, which was regarded as the fundamental constituent of matter, and the dynamics of the electron was derived from electromagnetism. Confrontation with experiment confirmed the theoretical prediction that the mass of the electron increased with its speed. This was an outstanding confirmation of the theory.

Since the dynamical properties of the electron could be derived from electromagnetism, and as the electron was regarded as the fundamental constituent of matter, many physicists thought that matter should be regarded as an electromagnetic phenomenon, and that all laws of matter (including mechanics) would be soon regarded as consequences of electromagnetism. This reduction of physics to electromagnetism was however shown to be impossible by Poincaré, who proved that it was necessary to introduce non-electromagnetic forces in the theory of the electron.

Notice that the development of the mass–velocity and the mass–energy relations depended on many distinct contributions by several different physicist. This is the rule, not the exception, in the history of science. To attribute a complex theory, such as relativity, to a single person, is a complete distortion of history.

Physicists usually praise Maxwell for his four equations (which he never wrote), and pardon him his belief in the ether (which was central to his work). We have seen, however, that the despised ether concept led to a series of dynamical studies that generated some of the most important results of the theory of relativity.

From Maxwell to Lorentz and Poincaré, the belief in the ether as the fundamental substratum of electromagnetic phenomena guided the study of its dynamical properties. Without that belief, the developments described in this chapter could not have occurred. Therefore, belief in the ether and the study of its properties was the fundamental step in the unfolding of relativistic dynamics.

Of course, confirmation of several consequences of the ether theory are not a proof that ether exists – just as the confirmation of several consequences of any theory (including Einstein's relativity or quantum mechanics) never prove that the theory is correct.

[24]This effect is relevant only when the body is submitted to a pressure or stress – independently of being accelerated – and when its volume is not neglected.

The theory that reached its peak at the hands of Lorentz and Poincaré was not Einstein's theory. Their world-view was different. They accepted the ether, although they also accepted that it was impossible to detect motion relative to this medium. Their epistemological approach was also different to Einstein's. However, the main predictions of Einstein's theory were already there, in the papers written before his first article. It seems impossible to distinguish, by any experiment, Lorentz's and Poincaré's theory from Einstein's special relativity. The empirical content of those theories is identical.[25,26]

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[25] There have been several claims, old and new, that Einstein's relativity theory and Lorentz's and Poincaré's ether theory can be experimentally contrasted in specific situations. Chalmers Sherwin [106] proposed and performed one such experiment, involving a fast rotating structure, and he maintained that it was incompatible with Lorentz's theory. I am not aware of any published criticism of Sherwin's experiment, but I would like to point out that some previously proposed experiments involved a series of mistakes that have been analysed by Torr and Kolen [119], by Rodrigues and Tiomno [96,97] and other authors.

[26] Editor's note: Lorentz's theory was valid only for "systems moving with any velocity less than that of light", as the title of his 1904 paper reads; in the text he pointed out that his theory did not have universal validity: "The only restriction as regards the velocity will be that it be less than that of light". This means that Lorentz did not rule out faster-than-light velocities, for signals or even for reference systems, but he only dealt with ordinary, 'subluminal' systems. On the other hand, in the final section of his 1905 paper Einstein explicitly says that, "Velocities greater than that of light have – as in our previous results – no possibility of existence". It seems to me that two theories such that one accepts, while the other denies, that faster-than-light systems and particles exist, cannot be considered to have "identical empirical content".

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Author's addition:

The editor of the book where this paper was published added a footnote (note # 26, page 44), criticizing one of my claims. As this note was added in a late editorial stage, I will take the opportunity of answering to the editor's remarks here.

The last paragraph of my paper reads:

> The theory that reached its peak at the hands of Lorentz and Poincaré was not Einstein's theory. Their world-view was different. They accepted the ether, although they also accepted that it was impossible to detect motion relative to this medium. Their epistemological approach was also different to Einstein's. However, the main predictions of Einstein's theory were already there, in the papers written before his first article. It seems impossible to distinguish, by any experiment, Lorentz's and Poincaré's theory from Einstein's special relativity. The empirical content of those theories is identical. (p. 44)

Concerning the last sentence, the editor remarked:

> Editor's note: Lorentz's theory was valid only for "systems moving with any velocity less than that of light", as the title of his 1904 paper states; in the text he pointed out that his theory did not have universal validity: "The only restriction as regards the velocity will be that it be less than that of light". This means that Lorentz did not rule out faster-than-light velocities, for signals or even for reference systems, but he only dealt with ordinary, 'subluminal' systems. On the other hand, in the final section of his 1905 paper Einstein explicitly says that, "Velocities greater than that of light have – as in our previous results – no possibility of existence". It seems to me that two theories (one accepting, the other denying that faster-than-light systems and particles exist) cannot be considered to have "identical empirical content". (p. 44, note 26)

Well, I cannot agree, for several reasons.

First: Lorentz did not state whether faster-than-light systems and particles exist. He only stated that the theory presented in his paper applied to speeds less than that of light. The editor's interpretation has no logical support.

Second: Even if Lorentz had stated that faster-than-light systems and particles can exist, I would maintain that "It seems impossible to distinguish, by any experiment, Lorentz's and Poincaré's theory from Einstein's special relativity. The empirical content of those theories is identical." Indeed, up to this day there is no experiment showing either that faster-than-light particles exist or that faster-than-light particles do not exist. Nowadays, it is impossible to distinguish BY EXPERIMENT the theory of relativity accepted by those who support the existence of tachyons and the theory of relativity accepted by those who deny the existence of tachyons and the decision between the two views cannot be empirical.

Roberto de Andrade Martins